Consistent Bimetric Theory and its Application to Cosmology

Angnis Schmidt-May

Department of Physics & The Oskar-Klein-Center
Stockholm University

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Based on

S.F. Hassan, R. Rosen, ASM
1109.3230

M. von Strauss, ASM, J. Enander, E. Mörtsell, S.F. Hassan
1111.1655

S.F. Hassan, ASM, M. von Strauss
1203.5283
1204.5202
1208.1515
1208.1797
1212.4525
1303.6940

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Philosophy

Approach I

1. invent a model to explain observations (e.g. extension of Standard Model with dark matter candidate, . . .)
   $\rightarrow$ many possibilities
2. check if the model is consistent, fits into a larger framework, has motivation besides cosmology, etc.

Philosophy

Approach I

1. invent a model to explain observations (e.g. extension of Standard Model with dark matter candidate, ...)
   $\rightarrow$ many possibilities
2. check if the model is consistent, fits into a larger framework, has motivation besides cosmology, etc.

Approach II

1. construct a consistent model guided by a fundamental question $\rightarrow$ few possibilities
2. check if it can solve (part of) an outstanding problem

Consistent field theories

spin 0:	Higgs boson	ϕ
spin $1/2$:	quarks, leptons	ψ^α
spin 1:	gluons, photons, W- & Z-bosons	A_μ

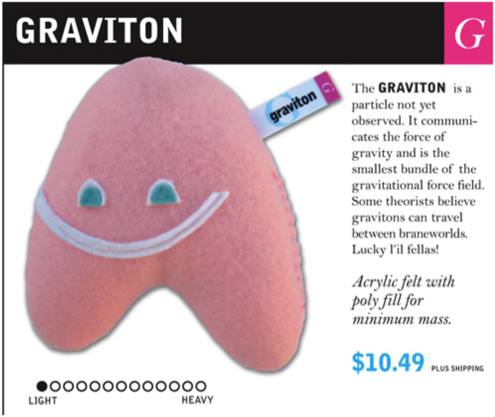
well-known consistent theories (classical and quantum!)

$\rightarrow$ **Standard Model of Particle Physics**

http://www.particlezoo.net

SPIN 2

$g_{\mu\nu}(x)$



General Relativity = classical theory for a **massless** spin-2 field.

What is the theory for a **massive** spin-2 field?

Massive gravity

- Massless theory = General Relativity with Einstein-Hilbert action for metric tensor $g_{\mu\nu}$

$$S_{\mathrm{EH}} = M_{\mathrm{P}}^2 \int \mathrm{d}^4x \sqrt{g}\, R(g)$$

- Mass term should have non-derivative interactions of $g_{\mu\nu}$
- Lorentz invariance $\Rightarrow$ need second metric to contract indices

$$g^{\mu\nu} f_{\mu\nu} = \mathrm{Tr}\,(g^{-1} f)$$

- Massive gravity action is of the form

$$S_{\mathrm{MG}} \;=\; S_{\mathrm{EH}} \;-\; \int \mathrm{d}^4x \sqrt{g}\, V(g^{-1} f)$$

Massive gravity

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What determines $f_{\mu\nu}$?

Massive gravity

nondynamical background metric $f_{\mu\nu}$, fixed by hand

$\rightarrow$ 5 d.o.f. [Fierz & Pauli, 1939]

Bimetric theory

dynamical $f_{\mu\nu}$, determined by its equation of motion

$\rightarrow$ 5 + 2 = 7 d.o.f. [Rosen, 1940; Isham, Salam & Strathdee,1971/77]

Bimetric action:

$$
\begin{aligned}
S_{gf} &= m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \\
&+ m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) - \int \mathrm{d}^4x \sqrt{g}\, V(g^{-1}f)
\end{aligned}
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\end{aligned}
$$

But there's a problem . . . **GHOSTS!**

The ghost problem

- Ghost $\equiv$ field with negative kinetic energy

$$\begin{aligned} \mathcal{L} &= (\partial_t \phi)^2 \cdots \qquad &\text{(healthy)} \\ \mathcal{L} &= -(\partial_t \phi)^2 \cdots \qquad &\text{(ghost)} \end{aligned}$$

- consequences: classical instabilities, negative probabilities in quantum theory $\rightarrow$ must be avoided!

The ghost problem

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- consequences: classical instabilities, negative probabilities in quantum theory $\rightarrow$ must be avoided!

Fierz & Pauli (1939):
ghost-free linear theory for massive spin-2

Boulware & Deser (1972):
ghost always present at the nonlinear level

No consistent nonlinear massive gravity?

The ghost-free theory

Development of nonlinear massive gravity

- construction of a candidate for the consistent theory of massive gravity with flat $f_{\mu\nu} = \eta_{\mu\nu}$

  [de Rham, Gabadadze & Tolley, 2010]

- generalization to arbitrary backgrounds and bimetric theory

  [Hassan & Rosen, 2011]

- shown to be free of the ghost in Hamiltonian analysis

  [Hassan & Rosen, 2011; Hassan, ASM & Rosen, 2011]

→ **First consistent nonlinear theory for massive spin-2 fields!**

Ghost-free interaction potential

$$V\left(g^{-1}f\right) \;=\; m^4 \sum_{n=0}^{4} \beta_n \, e_n\left(\sqrt{g^{-1}f}\,\right)$$

The potential involves ...

... an arbitrary mass scale m, 5 free parameters β_n

... the elementary symmetric polynomials

$$e_n(S) = \frac{2}{n!(4-n)!} \epsilon_{\mu_1 \ldots \mu_n \lambda_{n+1} \ldots \lambda_4} \epsilon^{\nu_1 \ldots \nu_n \lambda_{n+1} \ldots \lambda_4} S^{\mu_1}{}_{\nu_1} \ldots S^{\mu_n}{}_{\nu_n}$$

... a matrix square-root $\left(\sqrt{g^{-1}f}\,\right)^2 = g^{-1}f$

Summary: massive gravity vs. bimetric theory

$$S_{gf} = m_g^2 \int \mathrm{d}^4x \sqrt{g}\, R(g) \quad + m_f^2 \int \mathrm{d}^4x \sqrt{f}\, R(f) \\ - m^4 \int \mathrm{d}^4x \sqrt{g}\, \sum_{n=0}^{4} \beta_n e_n \left(\sqrt{g^{-1} f} \right)$$

- ghost-free massive gravity ($m_f = 0$)
  describes self-interactions of a massive spin-2 field in a fixed background

- ghost-free bimetric theory ($m_f \neq 0$)
  describes interactions of a massless and massive spin-2 field

Bimetric cosmology

Matter coupling

Simplest case: only one of the metrics couples to matter

$$
\begin{aligned}
S_{gf} = m_g^2 \int \mathrm{d}^4x\sqrt{g}\; R(g) \quad & + m_f^2 \int \mathrm{d}^4x\sqrt{f}\; R(f) \\
& - m^4 \int \mathrm{d}^4x\sqrt{g}\; \sum_{n=0}^{4} \beta_n e_n\left(\sqrt{g^{-1}f}\right) \\
& + \int \mathrm{d}^4x\sqrt{g}\; \mathcal{L}_{\mathrm{matter}}
\end{aligned}
$$

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The bimetric action gives rise to

- two sets of modified Einstein equations
- one Bianchi constraint

$\rightarrow$ derive classical solution, e.g. with homogeneous $\&$ isotropic ansatz for both metrics

Cosmological solutions

Homogeneous and isotropic ansatz:

$$\begin{aligned} T^g_{\mu\nu} &= \mathrm{diag}\big(-\rho, p, p, p\big) \\ g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu &= -\mathrm{d}t^2 + a(t)^2\Big(\mathrm{d}r^2 + r^2\mathrm{d}\Omega\Big) \\ f_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu &= -X(t)^2\mathrm{d}t^2 + Y(t)^2\Big(\mathrm{d}r^2 + r^2\mathrm{d}\Omega\Big) \end{aligned}$$

- solve Bianchi constraint for $X(t)$
- solve $f_{\mu\nu}$ e.o.m. for $Y(t)$
- $g_{\mu\nu}$ e.o.m. becomes modified Friedmann equation of the form

$$\left(\frac{\dot{a}}{a}\right)^2 = F[\rho(t)]$$

[Volkov 2011; von Strauss, ASM, Enander, Mörtsell, Hassan, 2011; Comelli, Crisostomi, Nesti, Pilo, 2011]

Fit to observational data

- viable cosmological solutions exist for different parameter choices [Akrami, Koivisto, Mota $\&$ Sandstad, 2013]
- no vacuum energy needed $\rightarrow$ self-acceleration
- Hubble scale set by mass parameter m and combination of interaction parameters
- background evolution very similar to ΛCDM
- distinguishable from ΛCDM by e.g. effective equation of state, growth index [Koennig $\&$ Amendola, 2014]

Perturbations

- around proportional backgrounds $f_{\mu\nu} = c^2 g_{\mu\nu}$ (maximally symmetric):

massless $\delta G_{\mu\nu} \propto m_g^2\, \delta g_{\mu\nu} + m_f^2\, \delta f_{\mu\nu}$

massive $\delta M_{\mu\nu} \propto \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}$ [Hassan, ASM & von Strauss, 2012]

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- around FRW backgrounds: **very difficult! :(**

  Problems:

  - identification of physical variables
  - solving a complicated system of differential equations

  [Comelli, Crisostomi & Pilo, 2012; Berg, Buchberger, Enander, Mörtsell & Sjörs, 2012]

Perturbation equations (only half of the full set!):

$$-\frac{1}{a^2}\nabla^2\Psi_g + 3H\left(H\Phi_g + \dot{\Psi}_g\right) + \frac{m^2YP}{2a^3}\left[3\left(-\Psi_f + \Psi_g - \frac{YK}{X}\mathcal{F}\right) + \nabla^2\mathcal{B}\right] = \frac{1}{2M_g^2}\delta T_0^0 \qquad (31)$$

$$-\partial_i\left(\dot{\Psi}_g + H\Phi_g\right) + \frac{m^2YXP}{2a\,(aX+Y)}\partial_i\left(\mathcal{F} + \frac{Y}{X}\dot{\mathcal{B}}\right) = \frac{\delta T_i^0}{2M_g^2} \qquad (32)$$

$$\begin{aligned}&\ddot{\Psi}_g + H\dot{\Phi}_g + 3H\left(H\Phi_g + \dot{\Psi}_g\right) + 2\dot{H}\Phi_g + \frac{1}{2a^2}\left(\partial_j^2 + \partial_k^2\right)\left(\Phi_g - \Psi_g\right) + \qquad (33)\\ &+\frac{m^2}{2a^2}\left\{P\left[X\left(\Phi_f - \Phi_g\right) - (Y\mathcal{F})^{\bullet}\right] + YQ\left[2\left(-\Psi_f + \Psi_g - \frac{YK}{X}\mathcal{F}\right) + \left(\partial_j^2 + \partial_k^2\right)\mathcal{B}\right]\right\} = \frac{1}{2M_g^2}\delta T_i^i\end{aligned}$$

$$-\frac{1}{2a^2}\partial^i\partial_j\left(\Phi_g - \Psi_g\right) - \frac{m^2YQ}{2a^2}\partial^i\partial_j\mathcal{B} = \frac{1}{2M_g^2}\delta T_j^i \qquad (34)$$

[Berg, Buchberger, Enander, Mörtsell & Sjörs, 2012]

$\Rightarrow$ predictions for CMB & structure formation still unclear

Extra symmetries (?)

A very special bimetric model [Hassan, ASM & von Strauss, 2012]

For a particular choice of interaction parameters

$$\beta_1 = \beta_3 = 0, \quad \alpha^4\beta_0 = 3\alpha^2\beta_2 = \beta_4$$

the theory seems to have additional symmetry properties:

- mass on Higuchi bound: $m_{\rm FP}^2 = \frac{2}{3}\Lambda$
  $\Rightarrow$ gauge redundancy for fluctuations around de-Sitter backgrounds $\rightarrow$ "partial masslessness" [Deser & Waldron, 2001]

- proportionality constant c not determined by equations
  $\Rightarrow$ nonlinear scaling symmetry of the background

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Is there evidence for a nonlinear gauge symmetry away from maximally symmetric backgrounds?

Relation to Conformal Gravity [Hassan, ASM & von Strauss, 2013]

- solve the $g_{\mu\nu}$ equation of motion for $f_{\mu\nu}$ and plug the solution into the $f_{\mu\nu}$ equation to obtain an effective equation for $g_{\mu\nu}$
  $\rightarrow$ relates bimetric theory to, e.g., New Massive Gravity

- for the model of the previous slide the result is:

$$B_{\mu\nu} + \mathcal{O}\left(\frac{R^3}{m^6}\right) = 0 \qquad \text{where } B_{\mu\nu} \text{ is the Bach tensor}$$

- to lowest order in derivatives, this is the equation of motion for Conformal Gravity

$$S_{\rm CG} \propto \int \mathrm{d}^4x\sqrt{g}\left(R^{\mu\nu}R_{\mu\nu} - \tfrac{1}{3}R^2\right)$$

action and equations of motion to lowest order in derivatives invariant under Weyl transformations: $g_{\mu\nu} \longmapsto \phi(x) g_{\mu\nu}$

Summary

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- classically consistent nonlinear theories:
  - massive gravity: 1 massive spin-2
  - bimetric theory: 1 massive $\&$ 1 massless spin-2
- bimetric models can reproduce ΛCDM cosmology at the background level without input of vacuum energy
- a particular bimetric model which is closely related to Conformal Gravity may exhibit a new type of gauge symmetry

Summary

- classically consistent nonlinear theories:
  - – massive gravity: 1 massive spin-2
  - – bimetric theory: 1 massive & 1 massless spin-2
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- a particular bimetric model which is closely related to Conformal Gravity may exhibit a new type of gauge symmetry

Open questions:

– perturbations?
– inflation?
– dark matter?
– quantum theory?

Back-up slides

Bimetric mass spectrum [Hassan, ASM & von Strauss, 2012]

proportional backgrounds: $f_{\mu\nu} = c^2 g_{\mu\nu}$

- gives two copies of Einstein's equations ($\alpha \equiv m_f/m_g$)

$$R_{\mu\nu}(g) - \frac{1}{2}g_{\mu\nu}R(g) + \Lambda_{g,f}(\alpha, \beta_n, c)g_{\mu\nu} = 0$$

- consistency condition: $\Lambda_g(\alpha, \beta_n, c) = \Lambda_f(\alpha, \beta_n, c)$ determines c

- fluctuations diagonalizable into mass eigenstates:

massless $\delta G_{\mu\nu} \propto \delta g_{\mu\nu} + \alpha^2 \delta f_{\mu\nu}$

massive $\delta M_{\mu\nu} \propto \delta f_{\mu\nu} - c^2 \delta g_{\mu\nu}$

with mass $m_{\mathrm{FP}} = m_{\mathrm{FP}}(\alpha, \beta_n, c)$

Partial masslessness [Deser & Waldron, 2001]

Equation of motion for massive fluctuation:

$$\bar{\mathcal{E}}^{\rho\sigma}_{\mu\nu}\delta M_{\rho\sigma} - \Lambda_g\left(\delta M_{\mu\nu} - \tfrac{1}{2}\bar{g}_{\mu\nu}\delta M\right) + \tfrac{1}{2}m^2_{\mathrm{FP}}\Big(\delta M_{\mu\nu} - \bar{g}_{\mu\nu}\delta M\Big) = 0\,,$$

For $m^2_{\mathrm{FP}} = \frac{2}{3}\Lambda_g$ (Higuchi bound) it has a gauge symmetry:

$$\Delta(\delta M_{\mu\nu}) = \bar{\nabla}_\mu\partial_\nu\phi(x) + \tfrac{\Lambda_g}{3}\bar{g}_{\mu\nu}\,\phi(x)$$

$\rightarrow$ removes one degree of freedom ("partially massless")