# Discovering Technicolor: Final Report for LHC School 2011 @ CP$^3$-Origins

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We provide a pedagogical introduction to a popular extension of the Standard Model (SM), the Minimal Walking Technicolor (MWT), in which the Higgs is composite. We therefore introduce the SM particle spectrum, review very briefly the electroweak dynamical symmetry breaking mechanism, and present the MWT salient features. After a brief review of collider physics and introduction to the Madgraph event generator, we finally present relevant MWT signatures at LHC.

## I. STANDARD MODEL

The Standard model (SM) has gauge symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$. In the SM there are three generations of fermions and five bosons (photon, gluon, and three gauge bosons) which are mediators of the corresponding interactions, as given in Figure I. Since

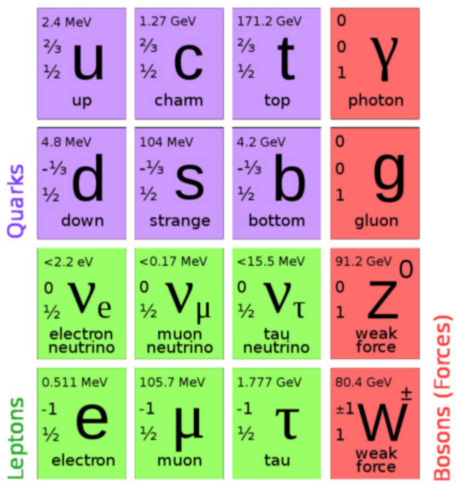



FIG. 1: Standard Model (SM) particle content

we know that the masses of the $Z$ and $W^{\pm}$ are not equal to zero than due to the Goldstones theorem we should have 3 broken generators among those generating the Electro Weak (EW) gauge transformations. Introducing a scalar Higgs boson can help us to give mass to the SM particles. Although no collider experimental result gives any deviation from SM prediction greater than $3\sigma$ (until lately, but there are 100s of measurements), there are still open problems:

- Fine tuning: the first 32 digits of the tree level and one loop contributions to MH must cancel to give MH $\approx$ 100 GeV
- Dark matter and energy: the SM describes only 4% of the Universes particle content
- Neutrino masses: the SM assumes the neutrino massless; allowed Yukawa interactions would be un-naturally small
- Matter-antimatter asymmetry: the relative abundance of matter cannot be generated by SM physics
- Unication: a common origin of the observed forces would reduce the theory arbitrariness; unication is ruled out in the SM

## II. DYNAMICAL SYMMETRY BREAKING AND TECHNICOLOR

The main idea of technicolor is to replace the Higgs mechanism of EW symmetry breaking by introducing some new particles, techniquarks, which are gauged by an additional $SU(N)_{TC}$ group and are thus confined at some scale $\Lambda_{TC}$ which is chosen to be of the order of 3 TeV. In order to understand how that can be achieved, it is useful to start with considering a simple massless QCD with one quark generation $q = (u, d)$ than possesses a classical chiral $SU(2)_L \times SU(2)_R$ symmetry which is known to be dynamically broken down to the isospin $SU(2)_V$ symmetry due to non-vanishing vacuum expectation value of quark-antiquark condensate $< \bar{q}q >_{vac} \neq 0$. The axial current $j_5^{\mu a} = \bar{q}\gamma^\mu\gamma_5\frac{\tau^a}{2}q$ of the broken symmetry is then effectively associated with its Goldstone bosons – pions: $j_5^{\mu a} = f_\pi \partial^\mu \pi^a$, where $f_\pi$ turns out to be the experimentally measured pion decay constant due to the matrix element $< 0|j_5^{\mu a}(x)|\pi^b(q) >= -if_\pi p^\mu e^{-ipx}\delta^{ab}$.

These considerations still hold even if EW $SU(2)_L \times U(1)_Y$ gauge interaction is given to the massless QCD quarks, so after effectively replacing the Lagrangian terms containing the quark axial current with the corresponding pion axial current terms we are left with the following two-point vector-pion vertices: $L = \frac{g}{2} f_{\pi^+} W^+_\mu \partial^\mu \pi^+ + \frac{g}{2} f_{\pi^-} W^-_\mu \partial^\mu \pi^- + \frac{g}{2} f_{\pi^0} W^0_\mu \partial^\mu \pi^0 + \frac{g'}{2} f_{\pi^0} B_\mu \partial^\mu \pi^0 + ....$ It can be easily shown by re-summing all tree-level insertions of these vertices into the W and B propagators that the three vector bosons acquire masses, i. e. $M_{W^\pm} = \frac{g}{2} f_{\pi^\pm}$, $M_Z = \frac{g^2+g'^2}{4} f_{\pi^0}$, $M_A = 0$, where $Z = (gW^3 - g'B)/\sqrt{g^2+g'^2}$ and $Z = (gB + g'W^3)/\sqrt{g^2+g'^2}$. Thus we can see that if only $f_\pi$ was 2650 times larger than its experimental value (93 MeV) then the non-vanishing quark-antiquark QCD condensate would eventually result in EW symmetry breaking with the correct masses for EW bosons! That is why it makes sense to introduce a new set of particles confined by a new gauge interaction at a scale $\Lambda_{TC}$.

However, in order to give masses to the SM fermions in absence of an elementary Higgs, TC has to be generalized to Extended TC which has a larger gauge group $SU(N_{ETC})$ which dynamically breaks down to $SU(N_{TC})$ at a scale $\Lambda_{ETC} >> \Lambda_{TC}$ effectively producing SM mass terms via four-fermion diagrams in which two legs belong to the confined technifermions. In QCD-like ETC the lowest breaking scale $\Lambda_{ETC}$ would be determined by the heaviest SM mass (of the t-quark), so $\Lambda_{ETC} \approx 10$ TeV. On the other hand, ETC will be producing flavor changing neutral currents which are known from experiment to be suppressed, giving $\Lambda_{ETC} > 1000$ TeV. One of the ways to reconcile these two facts is to have the so called Walking TC which has not fully QCD-like scale-behavior, but a broad range of scales in which the coupling constant changes very slowly. That can be achieved if the coupling constant has a suitable (approximate) infrared fixed point. TC theories with different number of technicolors and techniflavors were examined for the walking behavior and simplest one is known to be the Minimal Walking Technicolor (MWT).

## III. MINIMAL WALKING TECHNICOLOR

The Minimal Walking Technicolor (*minimal*: smallest content of fermion matter) is an extension of the Standard Model of the Electroweak Interactions (SM) gauged under the $SU(2)_{TC} \times SU(3)_C \times SU(2)_L \times U(1)_Y$ symmetry. It possesses four technifermions in the adjoint representation of the $SU(2)_{TC}$ gauge group: a left-handed techniquark doublet and two right-handed techniquark singlets:

$$Q_L^a = \begin{pmatrix} U_L^a \\ D_L^a \end{pmatrix},\ U_R^a,\ D_R^a \tag{1}$$

where $a \in \{1,2,3\}$ denotes the techniquark family. To remove the Witten anomaly [2], it is necessary to add a fourth fermion family, weakly charged but a singlet under technicolor:

$$L_L = \begin{pmatrix} N_L \\ E_L \end{pmatrix},\ N_R,\ E_R. \tag{2}$$

Gauge anomalies are cancelled with the following hypercharge assignment:

$$\begin{aligned} Y(Q_L) &= \frac{y}{2},\ Y(U_R, D_R) = \left(\frac{y+1}{2}, \frac{y-1}{2}\right), \\ Y(L_L) &= -\frac{3}{2}y,\ Y(N_R, E_R) = \left(\frac{-3y+1}{2}\right). \end{aligned} \tag{3}$$

Due to the assignment $Q = T^3 + Y$ with $T^3$ being the weak isospin, one obtains the SM hypercharge from Eq. (3) for $y = 1/3$ and the presence of a technigaugino for $y = 1$. The full Lagrangian of Minimal Walking Technicolor Theory then contains the Standard Model Lagrangian (without the Higgs part) and in addition Technicolor terms as follows:

$$\begin{aligned} \mathcal{L} &= \mathcal{L}_{SM} - \mathcal{L}_H + \mathcal{L}_{TC}, \\ \mathcal{L}_{TC} &= -\frac{1}{4}\mathcal{F}^a_{\mu\nu}\mathcal{F}^{a\mu\nu} + i\bar{Q}_L\gamma^\mu D_\mu Q_L + i\bar{U}_R\gamma^\mu D_\mu U_R + i\bar{D}_R\gamma^\mu D_\mu D_R \\ &\quad + i\bar{L}_L\gamma^\mu D_\mu L_L + i\bar{E}_R\gamma^\mu D_\mu E_R + i\bar{N}_R\gamma^\mu D_\mu N_R \end{aligned}$$

where $\mathcal{F}^a_{\mu\nu} = \partial_\mu\mathcal{A}^a_\nu - \partial_\nu\mathcal{A}^a_\mu + g_{TC}\epsilon^{abc}\mathcal{A}^b_\mu\mathcal{A}^c_\nu$ is the field strength tensor for techni-gauge bosons $\mathcal{A}_\mu$ ($a, b, c \in \{1,2,3\}$) and the left-handed covariant derivative is

$$D_\mu Q_L = \left(\delta^{ac}\partial_\mu + g_{TC}\epsilon^{abc}\mathcal{A}^b_\mu - i\frac{g}{2}\vec{W}_\mu \cdot \vec{\tau}\delta^{ac} - ig'\frac{y}{2}B_\mu\delta^{ac}\right) Q_L^c. \tag{4}$$

Note that in the above formula $\vec{W}_\mu$ and $B_\mu$ are the gauge bosons of the Standard Model, $\tau^a$ are Pauli matrices and $\epsilon^{abc}$ is the fully antisymmetric tensor.

Electroweak symmetry breaking is obtained by means of techniquark condensation, anal-

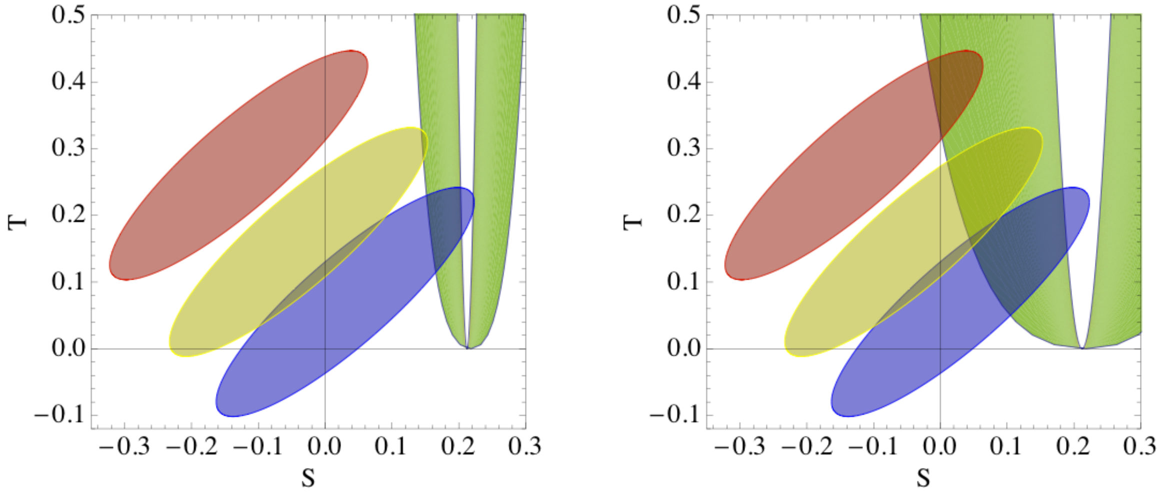



FIG. 2: The ellipses denote the experimental values of the $S$ and $T$ parameters at 90% confidence with $m_H \in \{117\ \text{GeV}, 300\ \text{GeV}, 1\ \text{TeV}\}$ – from lower to higher ellipses – while the values from the theory containing the fourth generation of leptons are shown in the green curve for $y = 1/3$ (left panel) and $y = 1$ (right panel).

ogously to the QCD case: $\langle \bar{Q}_i^\alpha Q_j^\beta \epsilon_{\alpha\beta} E^{ij} \rangle = -2\langle \bar{U}_R U_L + \bar{D}_R D_L \rangle \neq 0$ with

$$Q = \begin{pmatrix} U_L \\ D_L \\ -i\sigma^2 U_R^* \\ -i\sigma^2 D_R^* \end{pmatrix} \tag{5}$$

a four dimensional vector containing the already mentioned techniquark fields, arranged in the Weyl basis and transforming according to the fundamental representation of the $SU(4)$ group, $\epsilon_{\alpha\beta} = -i\sigma^2_{\alpha\beta}$ and

$$E = \begin{pmatrix} 0 & \mathbb{1}_2 \\ \mathbb{1}_2 & 0 \end{pmatrix} . \tag{6}$$

Results for $S$ and $T$ parameters of the theory are shown in Figure III. The ellipses denote the experimental values of the $S$ and $T$ parameters at 90% confidence with $m_H \in \{117\ \text{GeV}, 300\ \text{GeV}, 1\ \text{TeV}\}$ – from lower to higher ellipses – while the values from the theory containing the fourth generation of leptons are shown in the green curve for $y = 1/3$ (left panel) and $y = 1$ (right panel). Note that the masses of the fourth-generation leptons are varied between $m_Z$ and $10 m_Z$.

At this point, the theory contains only fields at the fundamental level (i.e., partonic fields and no composites nor interactions of composite fields). However, it is possible to construct an effective low-energy Lagrangian for Minimal Walking Technicolor with technimesons that has the following form:

$$\mathcal{L} = \frac{1}{2}\mathrm{Tr}[D_\mu M D^\mu M^\dagger] - \mathcal{V}(M) + \mathcal{L}_{ETC} + \mathcal{L}_{vector}, \tag{7}$$

$$\mathcal{V}(M) = -\frac{m_M^2}{2}\mathrm{Tr}[MM^\dagger] + \frac{\lambda}{4}(\mathrm{Tr}[MM^\dagger])^2 + \lambda'\mathrm{Tr}[MM^\dagger MM^\dagger] - 2\lambda''[\det M + \det M^\dagger], \tag{8}$$

$$\mathcal{L}_{ETC} = \frac{m_{ETC}^2}{4}\mathrm{Tr}[MBM^\dagger B + MM^\dagger], \tag{9}$$

$$\mathcal{L}_{vector} = -\frac{1}{2}\mathrm{Tr}[\tilde{W}_{\mu\nu}\tilde{W}^{\mu\nu}] - \frac{1}{4}\mathrm{Tr}[B_{\mu\nu}B^{\mu\nu}] - \frac{1}{2}\mathrm{Tr}[F_{\mu\nu}F^{\mu\nu}] + m^2\mathrm{Tr}[C_\mu C^\mu]. \tag{10}$$

Fields contained in the above equations are composed from two techniquarks. The matrix $M$ contains the scalar Higgs field and its pseudoscalar partner as well as nine pseudoscalar Goldstone bosons and their scalar partners; it transforms under $SU(4)$ according to $M \to uMu^T$ [and $u \in SU(4)$]. Note that the electroweak symmetry breaking reduces the original gauge group $SU(4)$ to its maximal diagonal subgroup $SO(4)$ and thus one obtains nine pseudoscalar Goldstone bosons. Three of the Goldstone bosons become longitudinal degrees of freedom of the gauge bosons $W^\pm$ and $Z^0$; the mass of the remaining six Goldstone bosons is generated by the Extended Technicolor (ETC) term $\mathcal{L}_{ETC}$, Eq. (9) with $B = \mathrm{diag}(\mathbb{1}_2, -\mathbb{1}_2)$. The axial anomaly is described by the term $[\det M + \det M^\dagger]$; it also generates the mass of the chiral partner of the Higgs state. Let us also note that Eq. (8) yields $m_H^2 = 2m_M^2$.

Furthermore, vector bosons are described by the four-dimensional traceless Hermitian matrix $A^\mu = A^{\mu a}T^a$ where $T^a$ are the generators of $SU(4)$ and $A^\mu \to uA^\mu u^\dagger$ under group transformations; then the field strength tensor $F_{\mu\nu}$ in Eq. (10) is $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\tilde{g}[A_\mu, A_\nu]$ and the tensors $\tilde{W}_{\mu\nu}$ and $B^{\mu\nu}$ represent the usual field strength tensors of the electroweak gauge group (but the $\tilde{W}$ bosons do not correspond to the physical $W^\pm$ states - the physical states are originated in the mixing of the $\tilde{W}$ with the composite vectors from the Lagrangian). Also, $C_\mu = A_\mu - (g/\tilde{g})G_\mu$ with $gG_\mu = gW_\mu^a L^a + g'B_\mu Y$ where $L^a$ and $Y$ generate the left-handed and hypercharge transformations.

## IV. COLLIDER PHYSICS

As particles of the sub-atomic scale obey the laws of quantum mechanics, we cannot predict the outcome of a single experiment, but only make statements about the *probabilities* of the occurrence of different events. Therefore, collider physicist normally try to collect as much data as possible, in order to get the best possible statistics to be compared with theoretical predictions.

In physics, the probability of a certain event at a collision (where collision, in contrast to usual sense, does not necessarily means that there is interaction) is represented by the *cross section* $\sigma$, which is defined

$$\sigma = \frac{\text{no. of events} \cdot A}{N_A N_B}$$

where $A$ is the cross-sectional area of the two beams and $N_A, N_B$ is the number of particles in each beam. The cross sectional evidently has units of area, and can be thought of as an effective surface area of the colliding particles.

The cross section is related to the measurable quantities $R$ and $L$, which are, respectively, the rate of events of the type in question and the luminosity of the beams i.e the number of collisions generated per time in the collider. The connection is the simple form

$$R = \sigma \cdot L$$

which enables scientists to calculate $\sigma$ from observed $R$ and $L$. The cross section is intimately connected to theory. If we imagine two relativistic particles, $A$ and $B$, colliding, each with four momentum in the center of mass frame $(E, 0, 0, \pm E)$ and therefore a squared center of mass energy $E_{cm}^2 = 4E^2$, then the cross section for a certain event is given from the master formula

$$d\sigma = \frac{1}{E_{cm}^2}\left(\prod_{i=1}^{N} \frac{d^3p_i}{(2\pi)^3}\frac{1}{2E_i}\right) \cdot (2\pi)^4 \delta^{(4)}(p_A + p_B - \sum p_i) \cdot |\mathcal{M}(p_A, p_B \to \{p_i\})|^2$$

which apart includes the matrix element $\mathcal{M}$, which contains all possible processes being able to produce the specific outgoing particles, and hence also all coupling constants and relevant particles of the theory in question.

Given a specific theory, it should be possible to either analytically or numerically predict the cross sections at certain energy and specific range of outgoing momenta, and comparison to experimental data then gives insight in the validity of the theory.

Interactions between particles are usually induced by the exchange of a boson particle. This boson particle will present itself in the Feynman diagrams of quantum field theory as a propagator of the form where the loop polarization $\Pi(p^2)$ is included:

$$\frac{1}{p^2 - m^2 - \Pi(p2)}$$

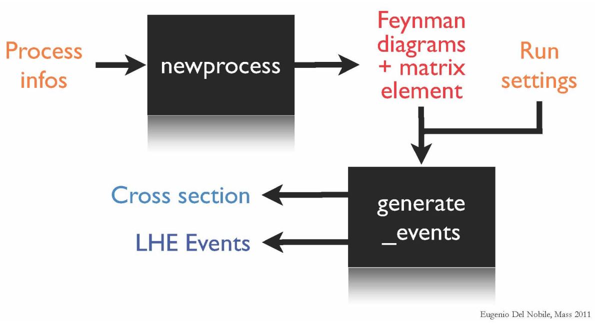
which evidently does not diverge but merely is maximum at $p^2 = m^2$. If terms in $\mathcal{M}$ with photon propagators and higher order terms are disregarded, then this would lead to a local maximum of the cross section at $p^2 = m^2$. This means that new particles should appear in experiments as an increased cross section at transferred momentum corresponding to the mass of the particle, also known as *Breit-Wigner resonances.*

## V. EVENT GENERATION IN MADGRAPH

For the tutorials we used an event generator called MadGraph. What an event generator in general is, is a software that on a given practical process produces a series of events, of that process. For example for the $pp \rightarrow l^+l^-$ process, Madgraph will produce all of the subprocesses. How Madgraph does this, and what we can use MadGraph for, we will discuss here.

What first comes to mind is that MadGraph has a minimal graphical user interface, what means that everything is done through the terminal and text documents. The overall design of Madgraph can be seen in this picture For a look into this we can see how to

FIG. 3: The main design of MadGraph




produce some of the results we used in our presentation. This first thing to do when using MadGraph, after installing it, is to define what model and process one want to analyze. To define these, we have to open the text file called proc_card.dat, in here we can change model to the SM, or Minimal Walking Technicolor, MWT (once this is implemented in MadGraph). It is also in proc_card.dat we can change which process we want to analyze, this is done simply by writing a process like $p\ p \rightarrow e^{+}\nu_{e}\ e^{-}\bar{\nu}_{e}$.

Now the process is defined, and with the command "new process" in the terminal, Madgraph calculates all the Feynman diagram and all the matrix elements for the sub-processes.

Ones we have does this we now need to calculate something with these diagrams, and that could be the cross section. To do this we need Something called a Particle Distribution Function, PDF, this just tells Madgraph how the momentum of the incoming particle is distributed among the elementary particles.

The PDF can be changed and this is done in the text document "run_card.dat". THe run card defines all the parameters for the command "generate_events", some examples are the beam energy, number of events and cuts, all can be interpreted as defining what collider we want to simulate. For our purpose we use these cuts;

- Beam energy 3500 GeV & 3500 GeV
- Transverse momentum $P_t > 15$ GeV to insure that an interaction happened
- Rapidity $\eta < 2,5$ to simulate the finiteness of the detectors

After this is done "generate_events" is used in the terminal, and MadGraph will generate events of the process, distributed according to the PDF.

Once this is done, we will in the "index.html" see all diagrams from the "new process" and the cross section from "generate_events". we can now from the events plot various things versus each other, for instance cross section versus the invariant mass. For this to work we need MadAnalysis, it will create plots based on events and the "plot_card.dat"-definitions. In "plot_card.dat" we can define plot ranges and a lot more.

One detail to be noted is that when creating plots, MadGraph cuts off some of the events if they are not in the plot range. This is a waste of computer power, since the events already have been generated, so if possible define ranges already in the "run_card-dat".

## VI. LHC SIGNALS

There are several ways of looking for MWT signals at LHC and perhaps one of the most promising chanels are looking for resonances in the invariant mass distributions corresponding to the two heavy composite vector boson triplets of MWT called $R_1^{\pm/0}$ (light) and $R_2^{\pm/0}$ (heavy) which mix the fundamental electroweak gauge fields $W^\mu$ and $B^\mu$ with the composite vector bosons $A^\mu$ arising from the condensation of the techniquarks. These mixed states can couple to the SM model fermions (see Figure 4) and can be seen as resonances in the invariant mass distribution of both leptonic and hadronic final states.

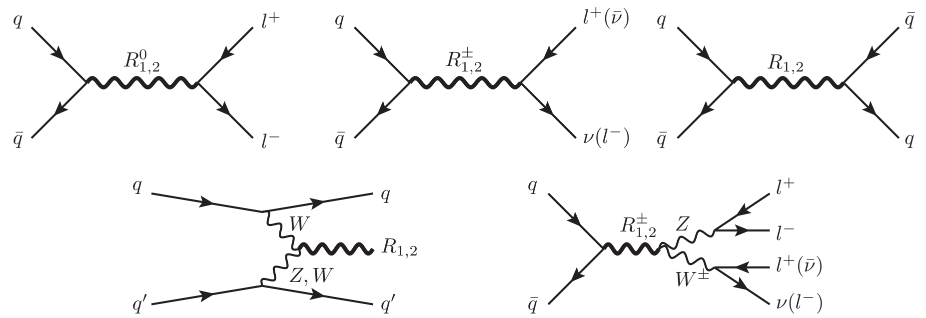



FIG. 4: tree level diagrams due to exchange of R bosons (with $\pm$ corresponding to $+(-)$ )

The standard model fermions do not couple directly to the TC vector bosons $A^\mu$ as can be seen from the lagrangian (neglecting QCD) for the SM fermions

$$\mathcal{L}_{\text{fermion}} = \bar{\Psi}_f \not{D} \Psi_f + \mathcal{L}(\Psi_f, M) \quad , \quad D_\mu = \partial_\mu + \frac{i}{2} g L^a W^a_\mu + \frac{i}{2} g' \Upsilon B_\mu$$

Where $M = \frac{1}{\sqrt{2}}\left(v + H + 2i\pi^a T^a\right)$ denotes the scalar content of MWT. The coupling is due to the mixing of the gauge fields that occur in the bosonic lagranigan

$$\mathcal{L}_{\text{boson}} = -\frac{1}{2}\operatorname{Tr}\left[W_{\mu\nu}W^{\mu\nu}\right] - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} - \frac{1}{2}\operatorname{Tr}\left[F_{\mu\nu}F^{\mu\nu}\right] + m^2 \operatorname{Tr}\left[C_\mu C^\mu\right] + \mathcal{L}(C_\mu, M)$$

$$F^{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\tilde{g}\left[A_\mu, A_\nu\right] \quad , \quad C_\mu = A_\mu - \frac{g}{\tilde{g}}\left(g L^a W^a_\mu + g' \Upsilon B_\mu\right)$$

When M aquires a VEV the bosonic lagrangian contains mixing matrices which gives rise to mass eigenstates corresponding to the ordinary SM bosons and the six new composite vector bosons $R^{\pm,0}_{1,2}$ which couple to the SM fermions through their lefthanded ( $W$ ) and hypercharge components ( $B$ ). The mixing is determined by the elctroweak part of the

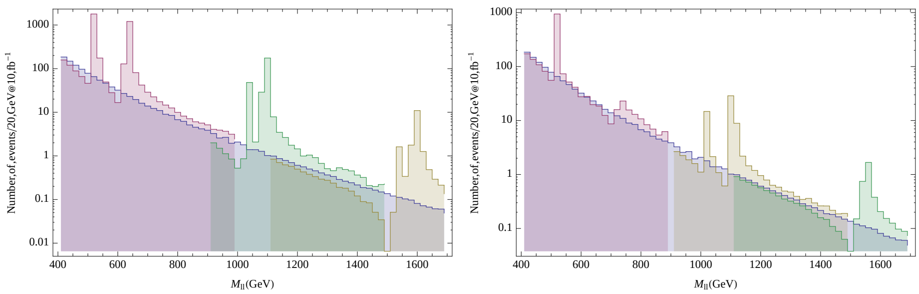



FIG. 5: left: Dilepton invariant mass distribution $M_{\ell\ell}$ for $\tilde{g} = 2$(left), 3(right) and $M_A = 0.5, 1, 1.5$ TeV: our results

$C$ fields and the coupling between the SM fermions and the R bosons are thus dependent on the ratio between $g$ and $\tilde{g}$

$$g_{R_{1,2}f\bar{f}} \sim \frac{g}{\tilde{g}}$$

From this we see that the coupling to the SM model is supressed as $\tilde{g}$ increases.

*Lepton pair production at LHC*

For our project we used MadGraph to simulate events of the type $p\bar{p} \to R^0_{1,2} \to \ell^+\ell^-$ with $\ell \in \{e, \mu\}$ at LHC with $\sqrt{s} = 7$ TeV and integrated luminosity of $10\ fb^{-1}$ for two different values of $\tilde{g} = 2, 3$ and $M_A = 0.5, 1, 1.5$ TeV with the other fundamental parameters chosen as in [1, chap.4 ]and compared it to the SM background wich we assumed to be due to $p\bar{p} \to \gamma, Z \to \ell^+\ell^-$ . We required that the leptons had $p_T > 15$ GeV and $|\eta| < 2.5$ and divided the work into three different groups each generating 50000 events for the different values of $M_A$ . We then plotted the invariant mass distribution of the two leptons for the combined events and compared it against the standard model background. The results are given in Figures 5 and 6 where we see that our results are in good agreement with the ones obtained in [1]. In Table I we show the masses and cross sections for the R resonances together with the integrated luminosity needed for 3 and $5\sigma$ detection. From the table we see that MWT for our choices of parameters is likely to be detected or ruled out by LHC, which is expected to reach $\mathscr{L} = 10\,fb^{-1}$ by 2012.

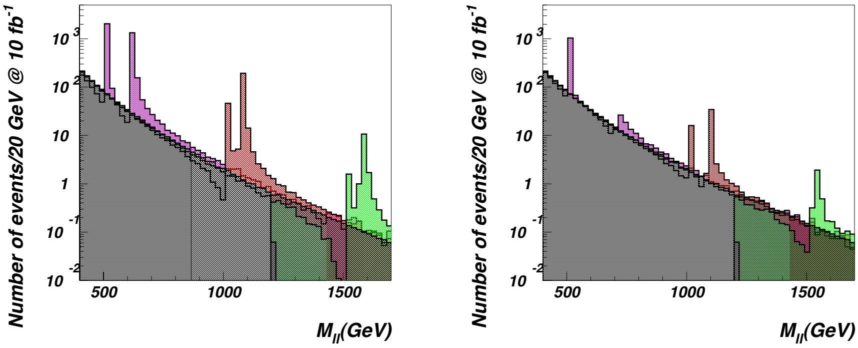



FIG. 6: left: Dilepton invariant mass distribution $M_{\ell\ell}$ for $\tilde{g} = 2(\text{left}), 3(\text{right})$ and $M_A = 0.5, 1, 1.5$ TeV: results given in [1]

TABLE I: $pp \to R_{1,2} \to \ell^+\ell^-$. Signal and background cross sections for $\tilde{g} = 2, 3$ and estimates for required luminosity for $3\sigma$ and $5\sigma$ signals. $M_{R_{1,2}}$ are the physical masses for the vector resonances in GeV.

| $\tilde{g}$ | $M_A$ | $M_{R_{1,2}}$ | $\sigma_S$ (fb) | $\sigma_B$ (fb) | $\mathscr{L}(\text{fb}^{-1})$ for $3\sigma$ | $\mathscr{L}(\text{fb}^{-1})$ for $5\sigma$ |
|---|---|---|---|---|---|---|
| 2 | 500 | $M_1 = 517$ | 194 | 3.43 | 0.012 | 0.038 |
| 2 | 500 | $M_2 = 623$ | 118 | 1.34 | 0.019 | 0.056 |
| 2 | 1000 | $M_1 = 1027$ | 4.57 | $9.17 \cdot 10^{-2}$ | 0.53 | 1.8 |
| 2 | 1000 | $M_2 = 1083$ | 16.4 | $5.60 \cdot 10^{-2}$ | 0.13 | 0.39 |
| 2 | 1500 | $M_1 = 1526$ | 0.133 | $5.91 \cdot 10^{-3}$ | 26 | 67 |
| 2 | 1500 | $M_2 = 1546$ | 0.776 | $2.81 \cdot 10^{-3}$ | 2.7 | 8.2 |
| 3 | 500 | $M_1 = 507$ | 93.5 | 3.71 | 0.037 | 0.090 |
| 3 | 500 | $M_2 = 715$ | 0.447 | 0.649 | 39 | 81 |
| 3 | 1000 | $M_1 = 1013$ | 1.32 | $8.81 \cdot 10^{-2}$ | 2.7 | 7.4 |
| 3 | 1000 | $M_2 = 1097$ | 2.94 | $5.15 \cdot 10^{-2}$ | 0.79 | 2.5 |
| 3 | 1500 | $M_1 = 1514$ | $3.19 \cdot 10^{-3}$ | $5.63 \cdot 10^{-3}$ | 6300 | 14000 |
| 3 | 1500 | $M_2 = 1541$ | 0.120 | $3.94 \cdot 10^{-3}$ | 29 | 68 |

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[2] E. Witten, Phys. Lett. B **117**, 324 (1982).