CKM matrix in models with dynamical mass generation

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Introduction

The Cabibbo-Kobayashi-Maskawa (CKM) matrix:

- is one of the main issues of the flavor physics
- is a 3×3 matrix ($3 =$ number of fermion generations)
- determines the strength of the charged current (i.e., $W^{\pm}$-mediated) interactions of different quarks
- there are tight experimental bounds on its value

$\Rightarrow$ desirable to be able to extract predictions for the CKM matrix from a given theory:

- in theories with condensing scalars (e.g., the Standard Model of electroweak interactions): No problem
- in theories with dynamical mass generation: Not so straightforward $\rightarrow$ topic of this talk

Plan of the talk

1. CKM matrix in the Standard Model
   A short review.

2. From mass matrices to self-energies
   Definition of dynamical mass generation for the purposes of this talk.

3. CKM matrix in models of dynamical mass generation

4. Summary

SM (1/3): quark mass Lagrangian (non-diagonalized)

How the subject is treated in the Standard Model (SM) – a review

- Consider 3 generations ('flavors') of up-type and down-type quarks:

$$u'_{L,R} = \begin{pmatrix} u'_1 \\ u'_2 \\ u'_3 \end{pmatrix}_{L,R} \qquad d'_{L,R} = \begin{pmatrix} d'_1 \\ d'_2 \\ d'_3 \end{pmatrix}_{L,R}$$

- Relevant part of the Yukawa interactions:

$$\mathcal{L}_{\text{Yukawa}} = \bar{u}'_R Y_u u'_L \phi_0 + \bar{d}'_R Y_d d'_L \phi_0 + \text{h.c.}$$

- SSB: $\phi_0 \to \langle \phi_0 \rangle \Rightarrow$ emergence of the mass Lagrangian:

$$\mathcal{L}_{\text{mass}} = -\bar{u}'_R \Sigma_u u'_L - \bar{d}'_R \Sigma_d d'_L + \text{h.c.}$$

where the *mass matrices* Σ_u, Σ_d are given by

$$\Sigma_u = -\langle \phi_0 \rangle Y_u \qquad \Sigma_d = -\langle \phi_0 \rangle Y_d$$

SM (2/3): quark mass Lagrangian (diagonalized)

Obtaining the quark masses:

- Σ_u, Σ_d – arbitrary complex 3×3 matrices in the flavor space $\Rightarrow$ can be diagonalized using the bi-unitary transformation:

$$\Sigma_u = U_u^\dagger M_u V_u \qquad \Sigma_d = U_d^\dagger M_d V_d$$

where

$$M_u = \mathrm{diag}(m_u, m_c, m_t) \qquad M_d = \mathrm{diag}(m_d, m_s, m_b)$$

and U_u, V_u, U_d, V_d are some unitary matrices

- Define new fields

$$\begin{aligned} u_L &= V_u u'_L & \qquad d_L &= V_d d'_L \\ u_R &= U_u u'_R & \qquad d_R &= U_d d'_R \end{aligned}$$

- $\Rightarrow$ the mass Lagrangian gets diagonalized:

$$\mathcal{L}_{\mathrm{mass}} = -\bar{u}_R M_u u_L - \bar{d}_R M_d d_L + \mathrm{h.c.}$$

SM (3/3): charged current interactions

Obtaining the CKM matrix:

- The charged current interactions in the original basis u', d'

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \bar{u}'_L \gamma^\mu d'_L W^\dagger_\mu + \text{h.c.}$$

  are flavor-diagonal.

- However, in the new, mass-diagonal basis u, d we obtain

$$\mathcal{L}_{\rm cc} = \frac{g}{\sqrt{2}} \bar{u}_L \gamma^\mu V_{\rm CKM} d_L W^\dagger_\mu + \text{h.c.}$$

  where

$$V_{\rm CKM} \equiv V_u V_d^\dagger$$

  is the wanted CKM matrix.

- Notice that $V_{\rm CKM}$ is unitary.

An idea of generalization

So far:

- the main input: *constant* matrices Σ
- two-fold meaning of Σ:
  - Σ living in the Lagrangian $\mathcal{L}_{\mathrm{mass}}$
  - Σ being part of the full propagator $\mathrm{i}S(p) = \int \mathrm{d}^4x \langle 0|T\psi(x)\bar{\psi}(0)|0\rangle \mathrm{e}^{\mathrm{i}p\cdot x}$:

$$S^{-1}(p) = \not{p} - \left(\Sigma P_L + \Sigma^\dagger P_R\right)$$

  where $P_{R,L} = (1 \pm \gamma_5)/2$.

- Σ typically generated by Yukawa couplings to some condensing scalar(s)

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Now - a generalization:

- the main input: *momentum-dependent* matrices $\Sigma(p^2)$
- only one meaning of $\Sigma(p^2)$:
  - $\Sigma(p^2)$ no more living in the Lagrangian
  - $\Sigma(p^2)$ is 1PI part of the propagator – the self-energy:

$$S^{-1}(p) = \not{p} - \left(\Sigma(p^2) P_L + \Sigma^\dagger(p^2) P_R\right)$$

- $\Sigma(p^2)$ typically generated dynamically, as a non-perturbative effect (see the next talk by A. Smetana . . .)

Quark self-energies

Let's assume that some unspecified mechanism generates the self-energies $\Sigma_u(p^2)$, $\Sigma_d(p^2)$:

- $\Sigma_u(p^2)$, $\Sigma_d(p^2)$ are 3×3 flavor matrices with a general momentum dependence
- $\Rightarrow$ the quark full propagators are

$$
\begin{aligned}
S_u^{-1}(p) &= \not{p} - \Big(\Sigma_u(p^2) P_L + \Sigma_u^\dagger(p^2) P_R\Big) \\
S_d^{-1}(p) &= \not{p} - \Big(\Sigma_d(p^2) P_L + \Sigma_d^\dagger(p^2) P_R\Big)
\end{aligned}
$$

- $\Rightarrow$ the quark masses given by the poles of the propagators:

$$
\begin{aligned}
\det\big[p^2 - \Sigma_u^\dagger(p^2)\,\Sigma_u(p^2)\big] &= 0 \\
\det\big[p^2 - \Sigma_d^\dagger(p^2)\,\Sigma_d(p^2)\big] &= 0
\end{aligned}
$$

The self-energies $\Sigma_u(p^2)$, $\Sigma_d(p^2)$ are momentum-dependent: $\Rightarrow$

- they cannot be interpreted as mass matrices
- one cannot perform the mass-diagonalization of the Lagrangian
- one cannot find V_{CKM} in the SM-like manner

$\rightarrow$ so how to proceed with V_{CKM} ?

Definition of CKM matrix

Recall the SM once again:

- The CKM matrix defined through the charged current interaction Lagrangian:

$$\mathcal{L}_{\mathrm{cc}} = \frac{g}{\sqrt{2}}\bar{u}_L\gamma^\mu V_{\mathrm{CKM}} d_L W^\dagger_\mu + \mathrm{h.c.} \qquad (1)$$

- For the process $\boxed{W^+ \to u_i + \bar{d}_j}$ this leads to the amplitude (in the leading order in g)

$$\mathcal{M} = \frac{g}{\sqrt{2}}\bar{u}_{u_i}(p)\gamma^\mu P_L (V_{\mathrm{CKM}})_{ij} v_{d_j}(k)\varepsilon_\mu(q) \qquad (2)$$

But now: we have self-energies $\Sigma_u(p^2)$, $\Sigma_d(p^2)$:

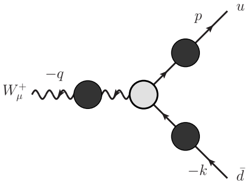
- The definition (1) of V_{CKM} is not possible.
- Idea: define V_{CKM} through the amplitude (2).

Indeed, the calculation of $\mathcal{M}$ is possible, let's see ...

How to calculate the amplitude $\mathcal{M}$

- Consider the connected 3-point function:

$$\mathrm{i}G_\mu(p,k,q) = \langle 0|T\, u'\, \bar{d}'\, W^+_\mu|0\rangle =$$



$$= S_u(p)\,\frac{g}{\sqrt{2}}\gamma^\nu P_L\, S_d(-k)\, D_{\mu\nu}(q)$$

- Then the amplitude $\mathcal{M}$ of the process $W^+ \to u_i + \bar{d}_j$ can be calculated using the Lehmann–Symanzik–Zimmermann (LSZ) reduction formula as

$$\mathrm{i}G_\mu(p,k,q) \xrightarrow[\substack{p^2\to m^2_{u_i}\\ k^2\to m^2_{d_j}\\ q^2\to M^2_W}]{} -\frac{\mathcal{U}_{u_i}(p)}{p^2-m^2_{u_i}}\,\frac{\bar{\mathcal{V}}_{d_j}(k)}{k^2-m^2_{d_j}}\,\frac{\varepsilon^*_\mu(q)}{q^2-M^2_W}\,\mathcal{M} + \ldots$$

where $\mathcal{U}_{u_i}$, $\mathcal{V}_{d_j}$ are multidimensional generalization of the usual u_{u_i}, v_{d_j}.

The result

The direct calculation gives

$$\mathcal{M} = \frac{g}{\sqrt{2}} \bar{u}_{u_i}(p)\gamma^\mu P_L (\tilde{V}_u \tilde{V}_d^\dagger)_{ij} v_{d_j}(k)\varepsilon_\mu(q)$$

where

$$\tilde{V}_u \equiv \begin{pmatrix} V_{u11}(m_u^2) & V_{u12}(m_u^2) & V_{u13}(m_u^2) \\ V_{u21}(m_c^2) & V_{u22}(m_c^2) & V_{u23}(m_c^2) \\ V_{u31}(m_t^2) & V_{u32}(m_t^2) & V_{u33}(m_t^2) \end{pmatrix}$$

$$\tilde{V}_d \equiv \begin{pmatrix} V_{d11}(m_d^2) & V_{d12}(m_d^2) & V_{d13}(m_d^2) \\ V_{d21}(m_s^2) & V_{d22}(m_s^2) & V_{d23}(m_s^2) \\ V_{d31}(m_b^2) & V_{d32}(m_b^2) & V_{d33}(m_b^2) \end{pmatrix}$$

and $V_u(p^2)$, $V_d(p^2)$ are defined as

$$\Sigma_u(p^2) = U_u^\dagger(p^2) M_u(p^2) V_u(p^2)$$
$$\Sigma_d(p^2) = U_d^\dagger(p^2) M_d(p^2) V_d(p^2)$$

with $M_u(p^2)$, $M_d(p^2)$ being diagonal and non-negative. Therefore we can identify

$$\boxed{V_{\rm CKM} = \tilde{V}_u \tilde{V}_d^\dagger}$$

Summary and outlook

What we have done:

- considered momentum-dependent generalization of mass matrices: the self-energies
- provided a model-independent algorithm how to arrive at an effective V_{CKM} in such a case
- shown that the V_{CKM} may be in the considered setup in principle non-unitary

What else can be done:

- consider also the electromagnetic and neutral current interactions
- consider also the mixing in the lepton sector with massive neutrinos
- consider also the mixing of Z and photon
- consider also the wave function renormalization

Reference: *Phys. Rev. D 81, 065029 (2010), arXiv:0904.0139*