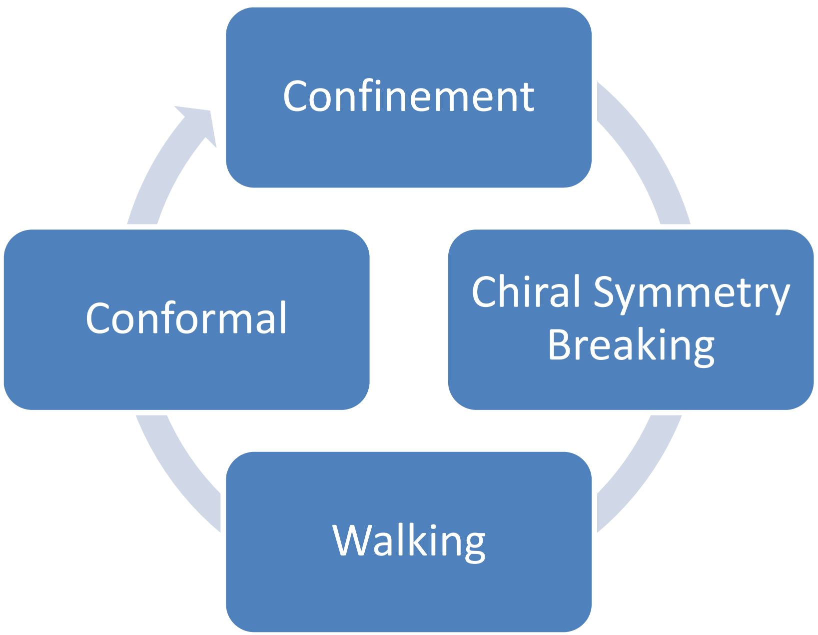
Recent Developments in TC/ETC Model Building

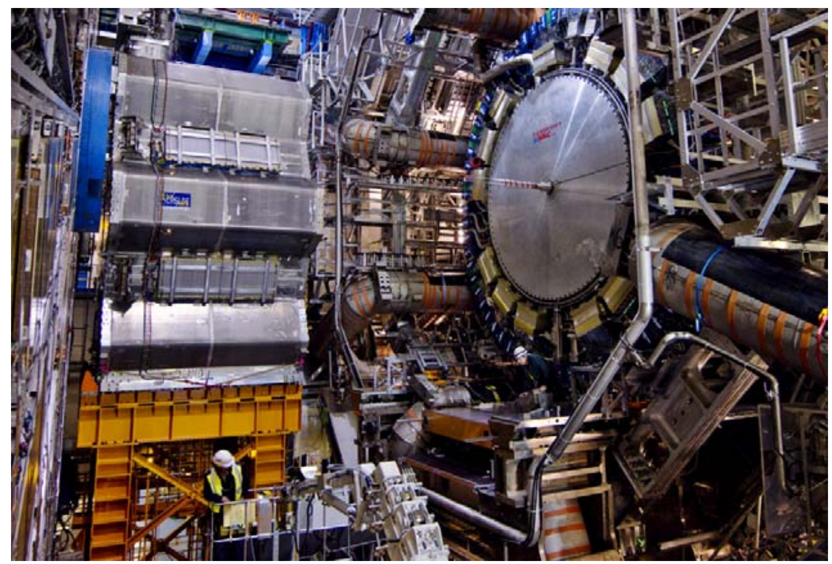
Thomas Ryttov
CP3 – Origins
Origin of Mass
May 7 – 2010

Part I & II

Phases of Gauge Theories



Technicolor and Extended Technicolor



Outline – Phases of Gauge Theories

– Single Representation

1. Ladder Approximation.
2. Lessons from SUSY: NSVZ beta function, unitarity bounds for conformal theories, Seiberg duality.
3. Propose an all-orders non-susy beta function.
4. Conformal window.
5. Duality and ’t Hoofts anomaly mathcing conditions

– Multiple representations

1. Conformal window -> conformal house
2. Use beta function for conformal house
3. Generalize Ladder Approximation to multiple representations

’t Hooft beta function

- Two loop beta function

$$\beta(g) = \frac{\beta_0}{(4\pi)^2} g^3 + \frac{\beta_1}{(4\pi)^4} g^5, \qquad \begin{cases} \beta_0 = \frac{4}{3} T(r) N_f - \frac{11}{3} C_2(G) \\ \beta_1 = \left(\frac{20}{3} C_2(G) + 4 C_2(r) \right) T(r) N_f - \frac{34}{3} C_2^2(G) \end{cases}$$

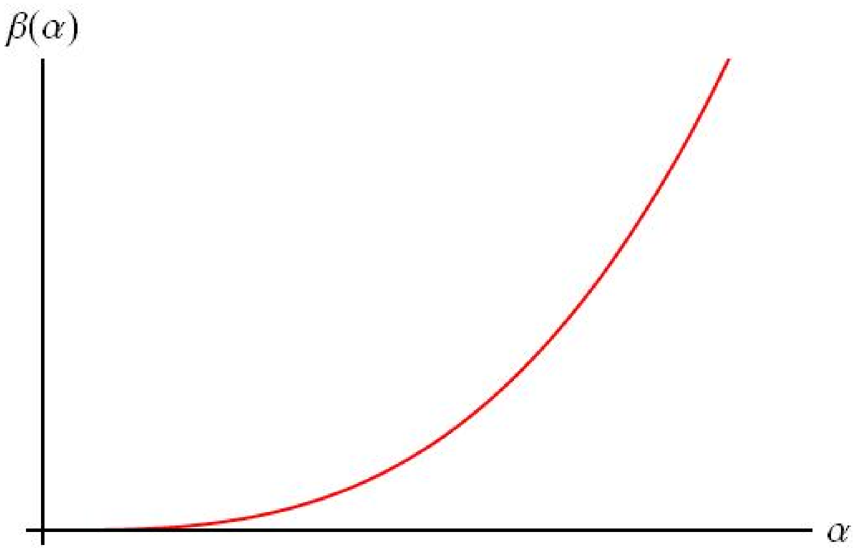
Group coefficients

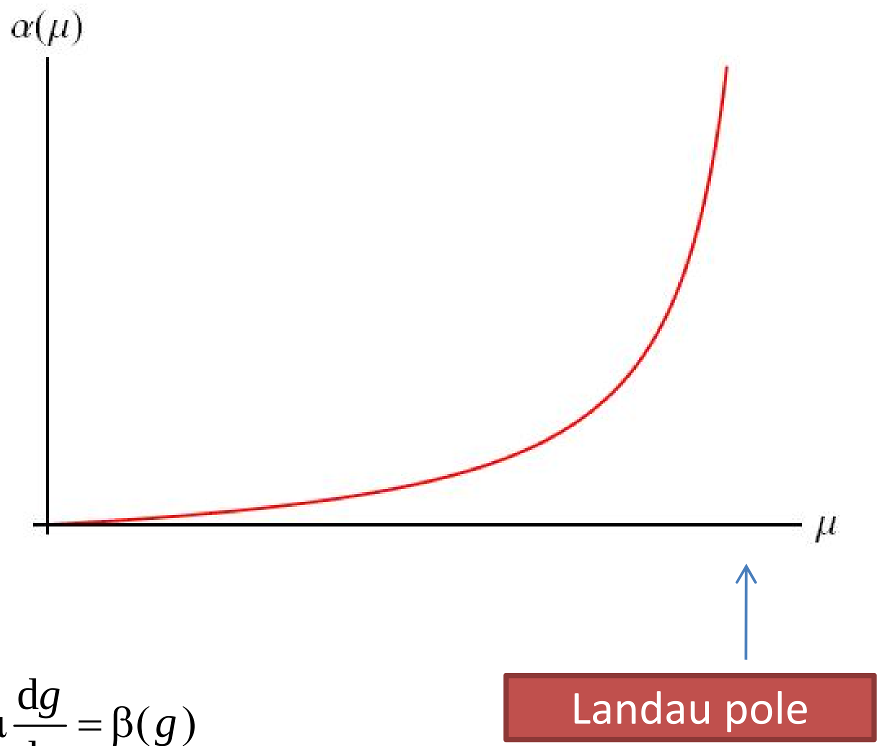
- ’t Hooft: Two first coefficents are universal.

- Physics depends on β_0 and β_1, and hence on colors, flavors and type of representation.

Non-Asymptotically Free

- Large N_f ($\beta_0 > 0$)

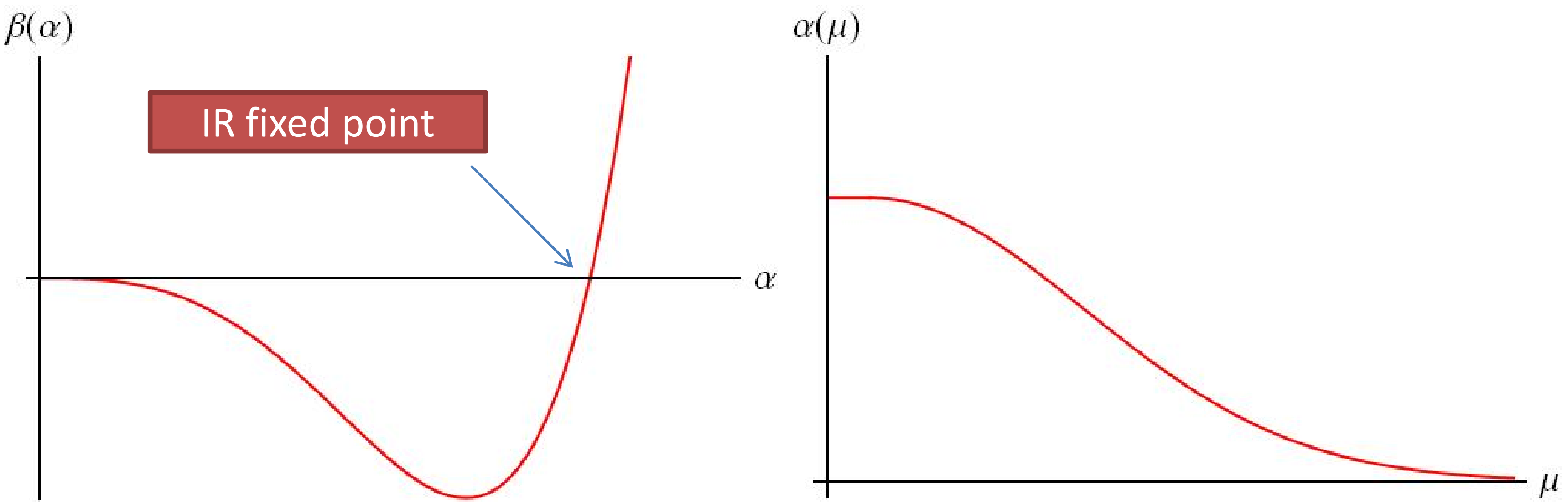




$$\alpha = \frac{g^2}{4\pi}, \qquad \mu \frac{dg}{d\mu} = \beta(g)$$

IR Conformal

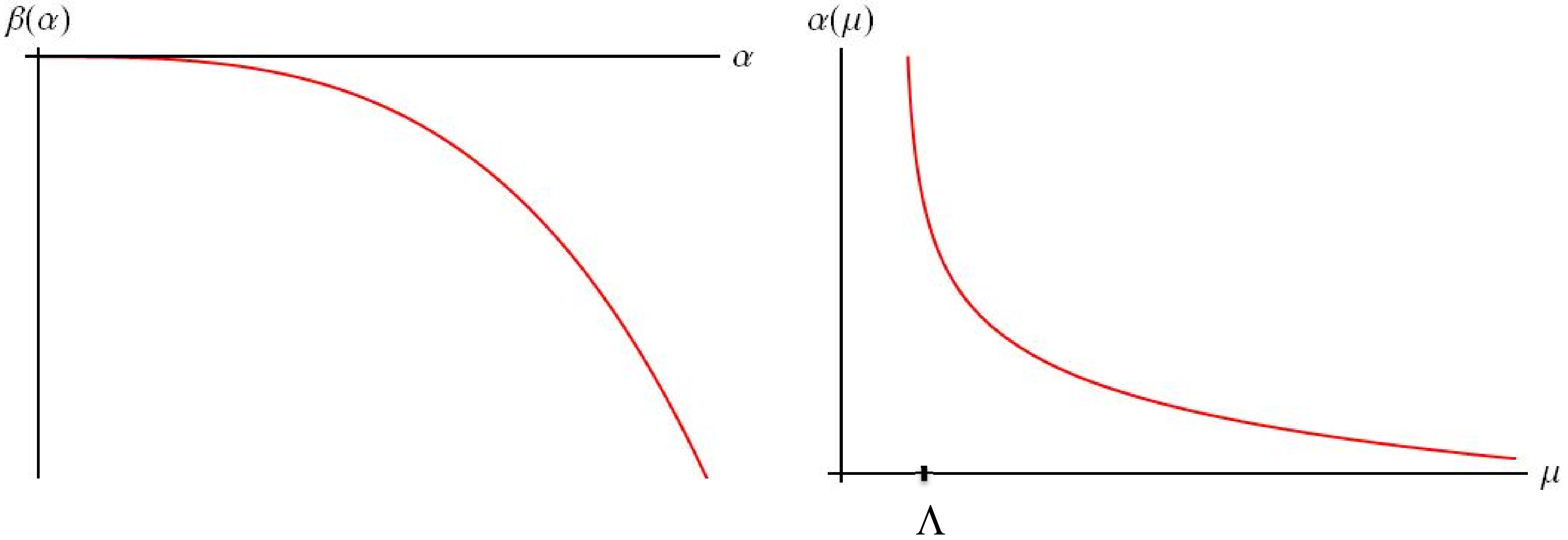
- Decrease N_f ($\beta_0<0$ and $\beta_1>0$)



$$\alpha = \frac{g^2}{4\pi}, \qquad \mu \frac{\mathrm{d}g}{\mathrm{d}\mu} = \beta(g)$$

QCD-like Theory

- Small N_f ($\beta_0<0$ and $\beta_1<0$)



$$\alpha = \frac{g^2}{4\pi}, \qquad \mu \frac{dg}{d\mu} = \beta(g)$$

Failure of perturbation theory

- Two loop fixed-point coupling

$$\alpha_* = -4\pi \frac{\beta_0}{\beta_1}$$

- α_* becomes large as $\beta_1 \to 0$
- Perturbation theory is inadequite.
- Chiral symmetry breaking could be triggered before α_* is reached.

Ladder Approximation

- The lowest order linearized nontrivial Schwinger-Dyson equation

$$\tilde{\Sigma}(p^2) = (3+\xi)\frac{C_2(r)}{4\pi}\int \frac{dk^2}{M^2}\frac{A(M)}{A(m)}\tilde{\Sigma}(k^2)\alpha, \qquad M = \max\left(p^2, k^2\right)$$

- With $A(p^2) = \left(\frac{\mu^2}{p^2}\right)^{\gamma}$ the massless renormalization group factor and $\gamma = \xi \frac{\alpha}{4\pi} C_2(r)$ the anomalous dimension of the field.

- Use scale invariant Ansatz $\tilde{\Sigma}\left(p^2\right) = \tilde{\Sigma}\left(\mu^2\right)\left(\frac{\mu^2}{p^2}\right)^b$

$$b\left(1-b\right) = \frac{1}{4}\frac{3C_2(r)}{\pi}\alpha + O(\alpha^2)$$

- Chiral symmetry breaking is associated to b changing real/comlex

- Critical value $\alpha_c = \frac{\pi}{3C_2(r)}$

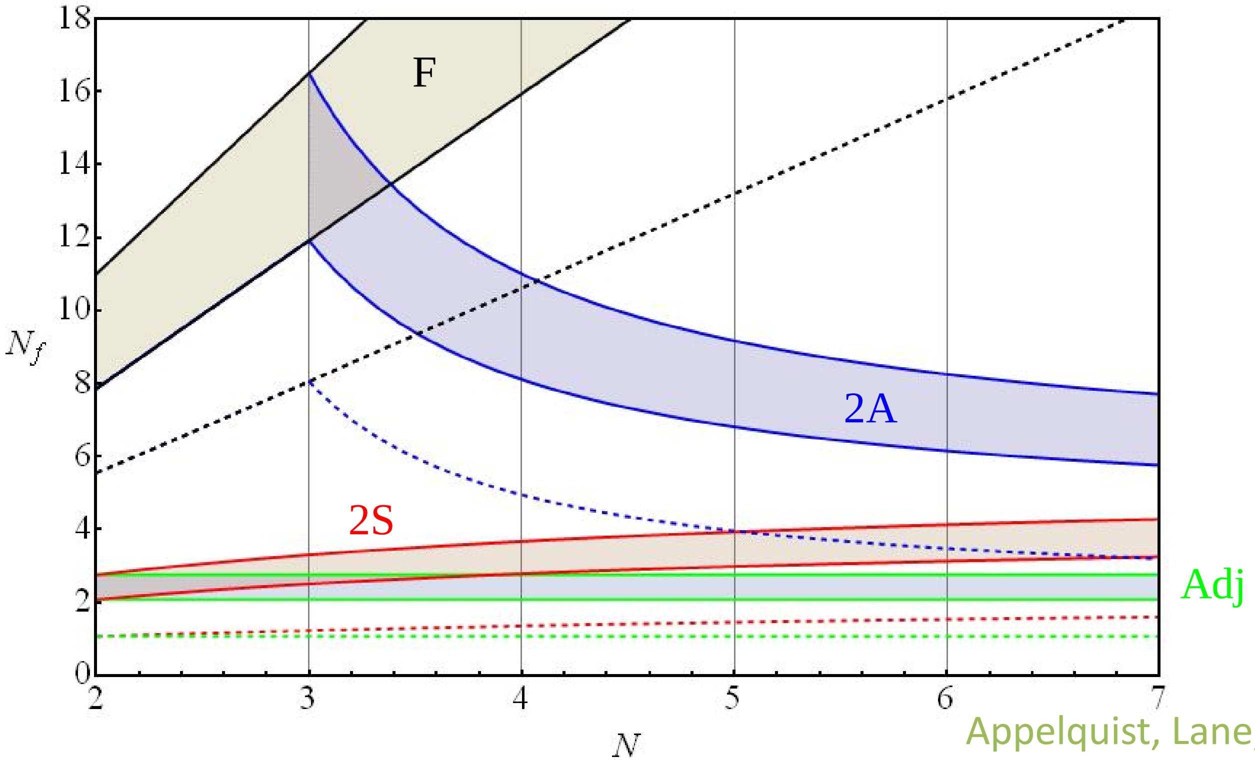
Ladder Approximation

- If $\alpha_c < \alpha_*$ we have chiral symmetry breaking.
- If $\alpha_c > \alpha_*$ we have IR conformal dynamics.
- Critical number of flavors

$$\alpha_c = \alpha_*$$

$$N_f = \frac{17C_2(G) + 66C_2(r)}{10C_2(G) + 30C_2(r)} \frac{C_2(G)}{T(r)}$$

Phase Diagram (ladder approximation)



Appelquist, Lane, Mahanta, Wijewardhana, Cohen, Georgi, Yamawaki, Shrock, Dietrich, Sannino, Tuominen

- Ladder approximation: Chiral symmetry breaking is triggered at

$$\gamma \sim 2b \sim 1$$

Lessons from SUSY

- Exact NSVZ beta function

Novikov, Shifman, Vainshtein, Zakharov

$$\beta(g) = -\frac{g^3}{(4\pi)^2} \frac{\beta_0 + 2T(r)N_f \gamma(g^2)}{1 - \frac{g^2}{8\pi^2} C_2(G)}$$

$$\gamma(g^2) = -\frac{g^2}{4\pi^2} C_2(r) + O(g^4)$$ ← Anomalous dimension

$$\beta_0 = 3C_2(G) - 2T(r)N_f$$ ← First beta function coefficient

- Note: relation between beta funtion and anomalous dimension.

- At zero of beta funtion

$$\frac{2}{3}T(r)N_f(1-\gamma) = C_2(G)$$

- Conformality requires

Mack

$$D(\Phi\tilde{\Phi}) = 2+\gamma \geq 1$$

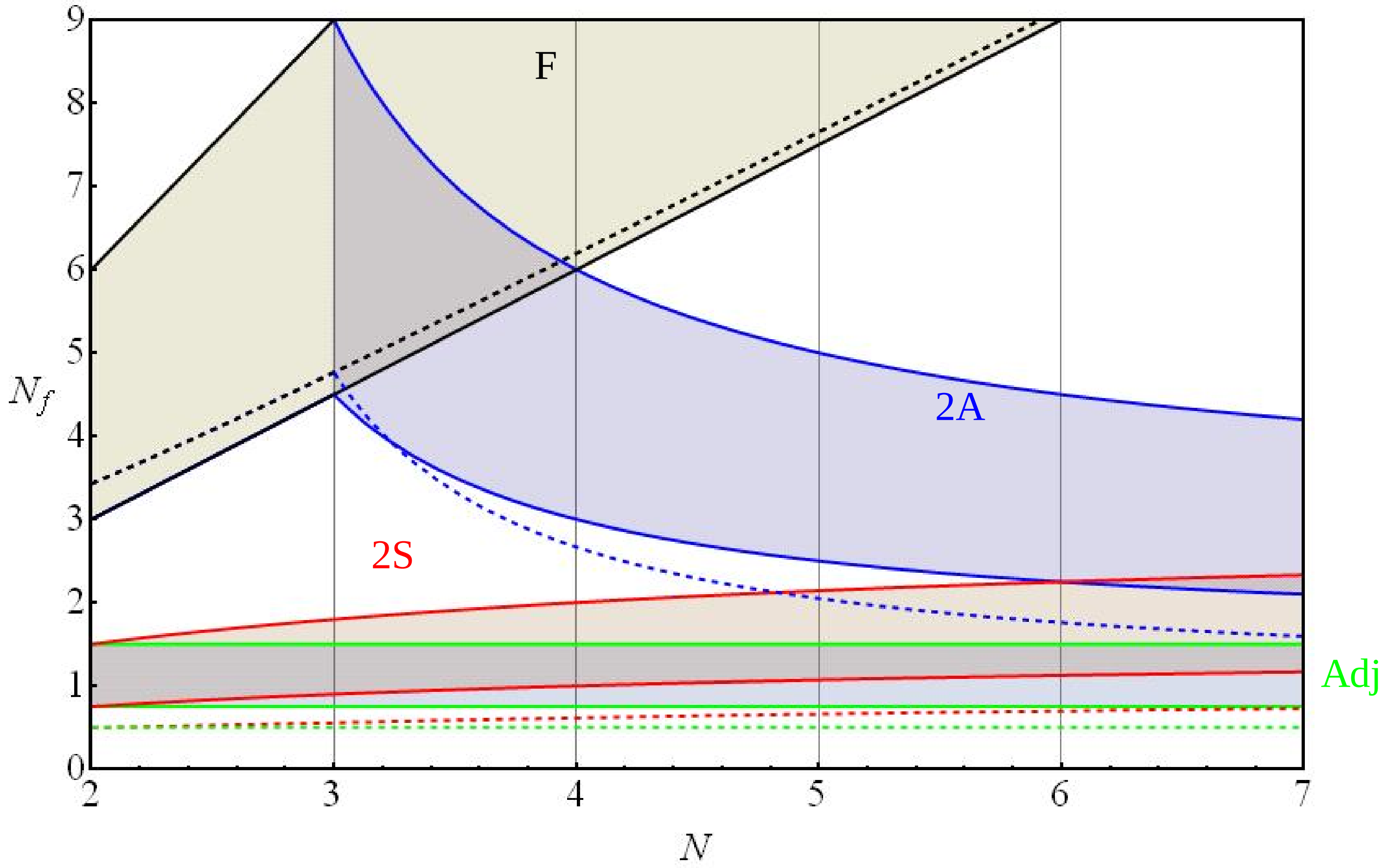
- Critical number of flavors

$$N_f^{critical} = \frac{3}{4}\frac{C_2(G)}{T(r)}$$

Seiberg

SUSY Phase Diagram

Seiberg, TR, Sannino



All-orders Beta Function Conjecture

$$\beta(g) = -\frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3}T(r)N_f\gamma(g^2)}{1 - \frac{g^2}{8\pi^2}C_2(G)\left(1 + \frac{2\beta_0'}{\beta_0}\right)}$$

TR, Sannino

$$\gamma(g^2) = \frac{3}{2}C_2(r)\frac{g^2}{4\pi^2} + O(g^4)$$

← Anomalous dimension

$$\beta_0 = \frac{11}{3}C_2(G) - T(r)N_f$$

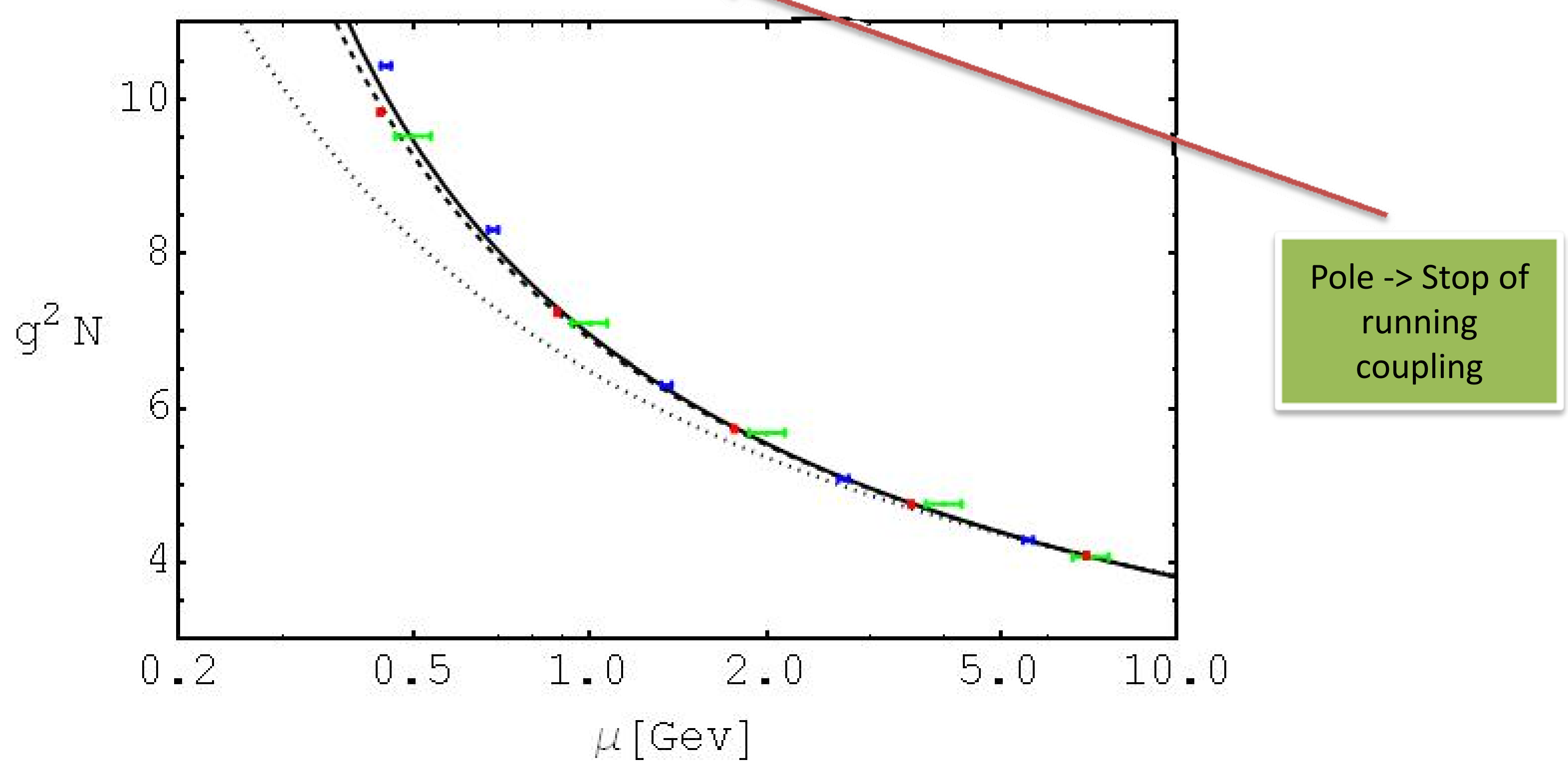
← First beta function coefficient

$$\beta_0' = C_2(G) - T(r)N_f$$

- Reduces to two-loop beta function for small coupling and has same form as NSVZ.

Yang-Mills for different N

- Yang-Mills: $$\beta_{YM}(g) = -\frac{g^3}{(4\pi)^2}\frac{\beta_0}{1-\frac{g^2}{(4\pi)^2}\frac{\beta_1}{\beta_0}}, \qquad \beta_0 = \frac{11}{3}N, \qquad \beta_1 = \frac{34}{3}N^2$$



- SU(2): Luscher, Sommer, Wolff, Weisz
- SU(3): Luscher, Sommer, Weisz, Wolff
- SU(4): Lucini, Moraitis

All-Orders: ———
2-Loops: - - - - -
1-Loop: ....

Bounds on Conformal Window

- Analysis similar to SUSY case:

  – At zero of beta function

$$\frac{2}{11}T(r)N_f(2+\gamma)=C_2(G)$$

  – Conformality requires

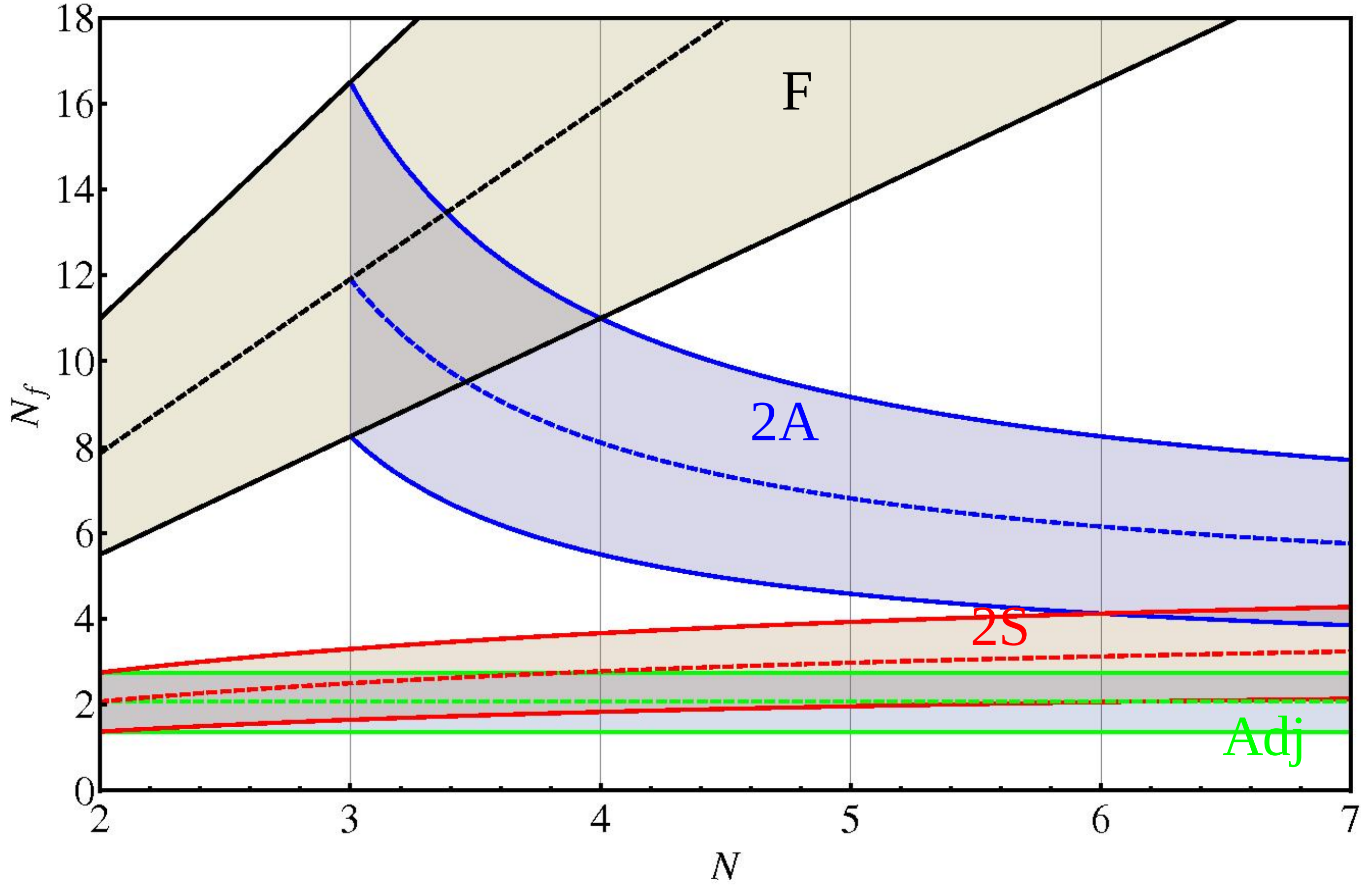
$$D(\bar{\psi}\psi)=3-\gamma\geq 1 \qquad \Rightarrow \qquad \gamma\leq 2$$

Mack

  – Critical number of flavors

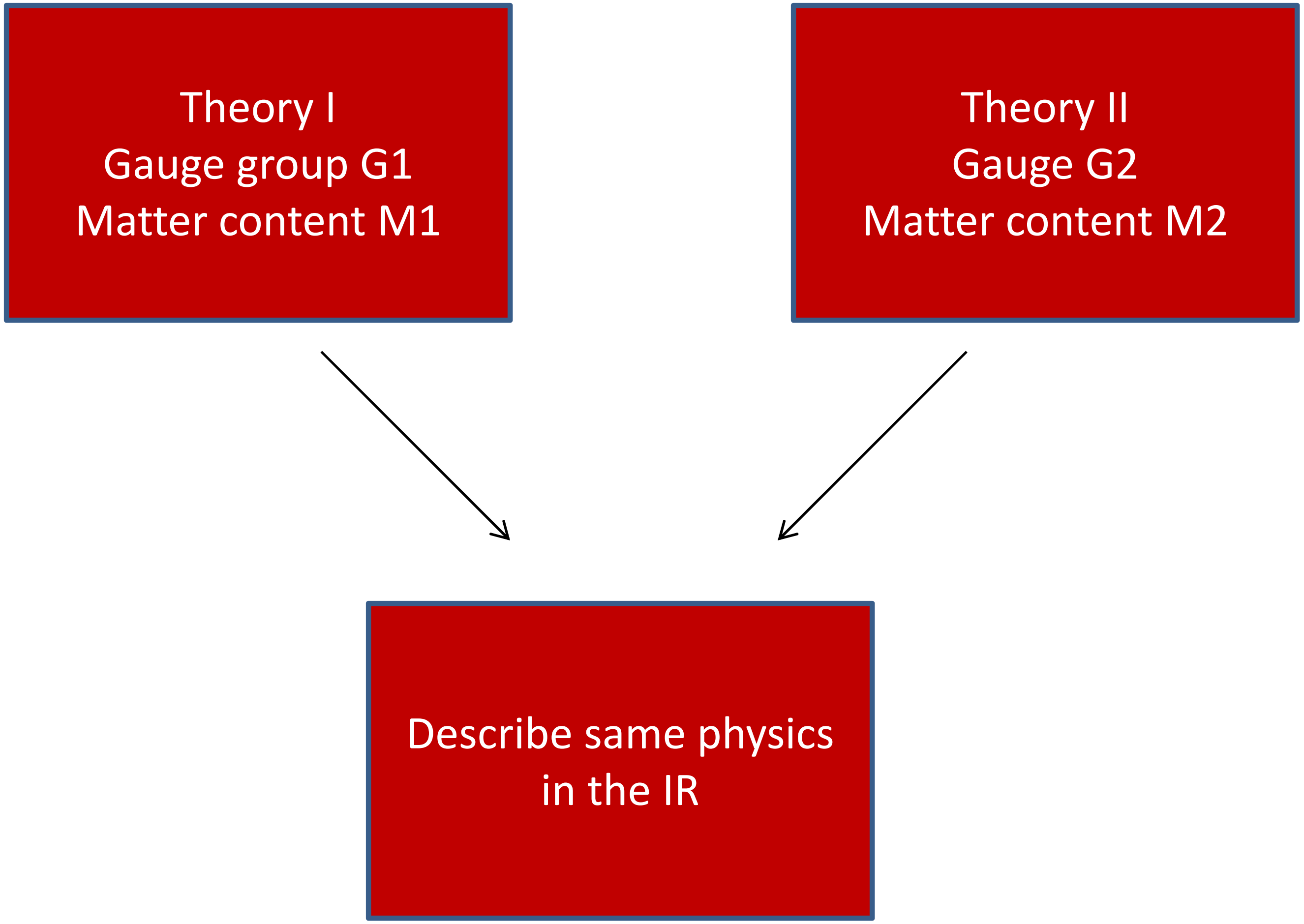
$$N_f=\frac{11}{8}\frac{C_2(G)}{T(r)}$$

Phase Diagram



- Compare to lattice simulations

Dualities


Theory I
Gauge group G1
Matter content M1
Theory II
Gauge G2
Matter content M2
Describe same physics
in the IR

- Establish a dual desrciption in the conformal window.

- Must satisfy ’t Hooft anomaly matching conditions

- Different gauge groups possible (redundancy)

- Different matter contents.

weak coupling <-----> strong coupling

Seiberg Dual

	[SU(N)]	$SU(N_f)_L$	$SU(N_f)_R$	U(1)	$U(1)_R$
Q	N	N_f	1	1	$\frac{N_f - N}{N_f}$
$\tilde{Q}$	$\bar{N}$	1	$\bar{N}_f$	-1	$\frac{N_f - N}{N_f}$

	$[SU(N_f\text{-}N)]$	$SU(N_f)_L$	$SU(N_f)_R$	U(1)	$U(1)_R$
q	$N_f - N$	$\bar{N}_f$	1	$\frac{N}{N_f - N}$	$\frac{N}{N_f}$
$\tilde{q}$	$\overline{N_f - N}$	1	N_f	$-\frac{N}{N_f - N}$	$\frac{N}{N_f}$
M	1	N_f	$\bar{N}_f$	0	$2\frac{N_f - N}{N_f}$

QCD Dual (Sannino)

	[SU(3)]	$SU(N_f)_L$	$SU(N_f)_R$	U(1)
Q	3	N_f	1	1
$\tilde{Q}$	$\overline{3}$	1	$\bar{N}_f$	-1

	[SU(2N$_f$-15)]	$SU(N_f)_L$	$SU(N_f)_R$	U(1)	# of copies
q	$2N_f - 15$	N_f	1	$\frac{3(2N_f-5)}{2N_f-15}$	1
$\tilde{q}$	$\overline{2N_f - 15}$	1	$\bar{N}_f$	$-\frac{3(2N_f-5)}{2N_f-15}$	1
A	1	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	1	3	2
$\tilde{A}$	1	1	$\overline{\begin{smallmatrix}\Box\\\Box\end{smallmatrix}}$	-3	2
B	1	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	$\Box$	3	-3
D	1	$\Box$	$\begin{smallmatrix}\Box\\\Box\end{smallmatrix}$	3	2

Multiple Representations

- Why study multiple representations?

- Construct walking technicolor models with minimal naive S parameter and smallest possible number of fermions. (Ultra Minimal Technicolor)

TR, Sannino

- Other famous examples:

1) SU(5) grand unified theory: $\bar{5}+10$ Georgi, Glashow

2) MSSM: Adj +F

Beta Function – Multiple Representation

$$\beta(g) = -\frac{g^3}{(4\pi)^2} \frac{\beta_0 - \frac{2}{3}\sum_{i=1}^{k} T(r_i) N_f(r_i)\gamma_i(g^2)}{1 - \frac{g^2}{8\pi^2} C_2(G)\left(1 + \frac{2\beta_0'}{\beta_0}\right)}$$

TR, Sannino

$$\gamma_i(g^2) = \frac{3}{2} C_2(r_i) \frac{g^2}{4\pi^2} + O(g^4)$$

Anomalous dimensions

$$\beta_0 = \frac{11}{3} C_2(G) - \sum_{i=1}^{k} T(r_i) N_f(r_i)$$

First beta function coefficient

$$\beta_0' = C_2(G) - \sum_{i=1}^{k} T(r_i) N_f(r_i)$$

Bounds on Conformal House

– At zero of beta function

$$\sum_{i=1}^{k} \frac{2}{11} T(r_i) N_f\left(r_i\right)(2+\gamma_i) = C_2(G)$$

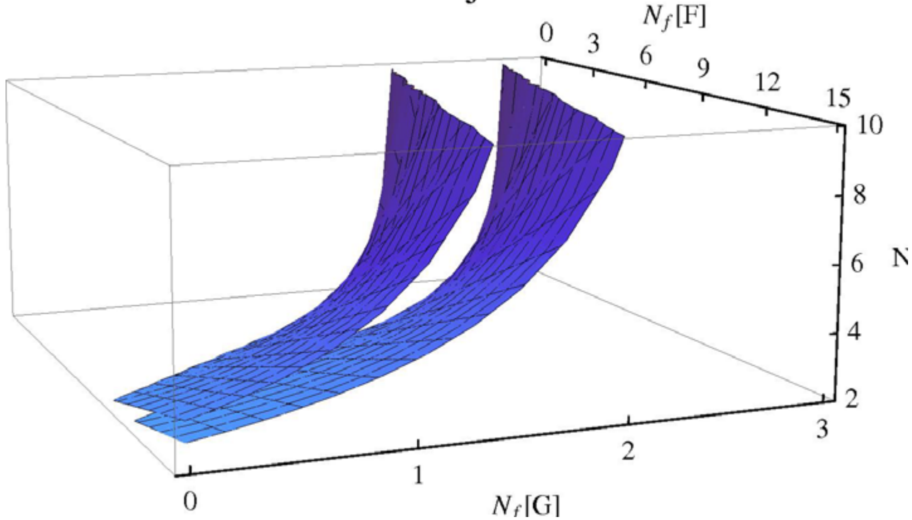
– Conformality requires

$$D(\overline{\psi}_i \psi_i) = 3 - \gamma_i \geq 1 \qquad \Rightarrow \qquad \gamma_i \leq 2$$

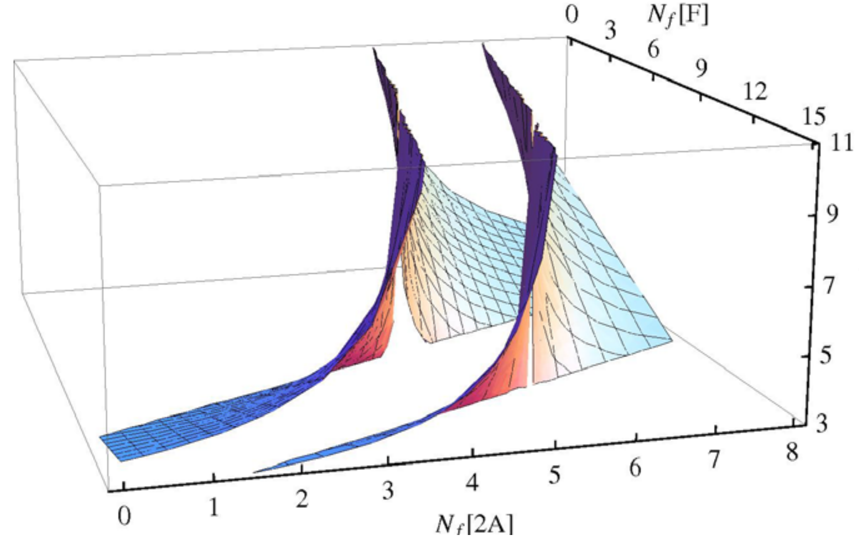
– Critical number of flavors

$$\sum_{i=1}^{k} \frac{8}{11} T\left(r_i\right) N_f\left(r_i\right) = C_2\left(G\right)$$

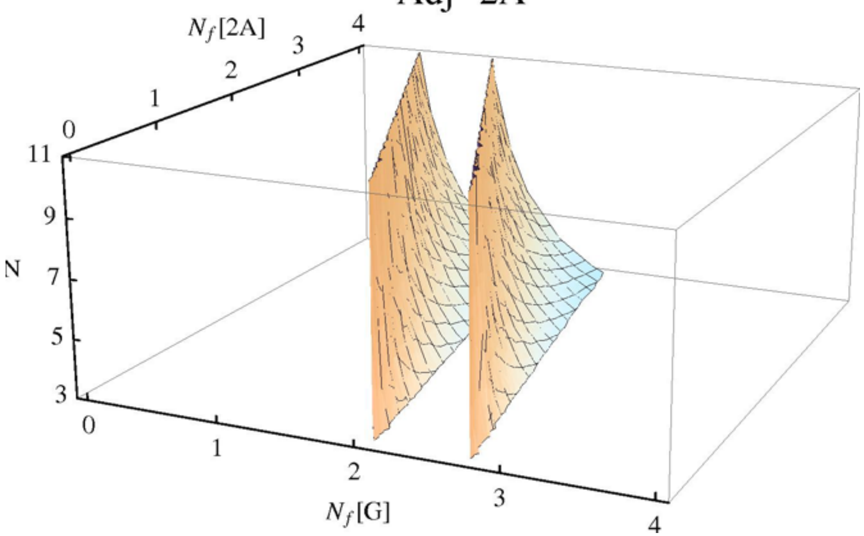
F−Adj



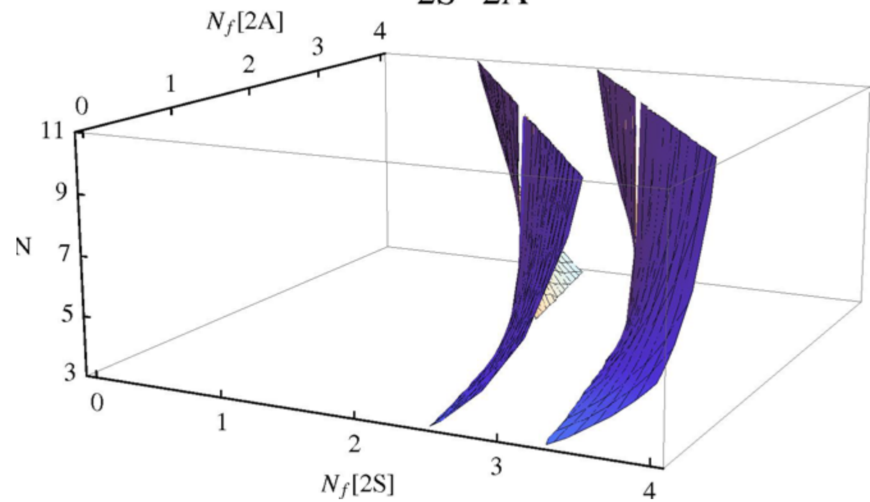
F−2A



Adj−2A



2S−2A



Conformal House in the Ladder App.

- There is a critical coupling associated to the triggering of each of the condensates

TR, Shrock (to be published)

$$\alpha_c(r_1) = \frac{\pi}{3C_2(r_1)} \qquad \alpha_c(r_2) = \frac{\pi}{3C_2(r_2)}$$

- At higher orders the kernel and the anomalous dimension of the fields will depend on the number of flavors of both representations and the gauge parameter.

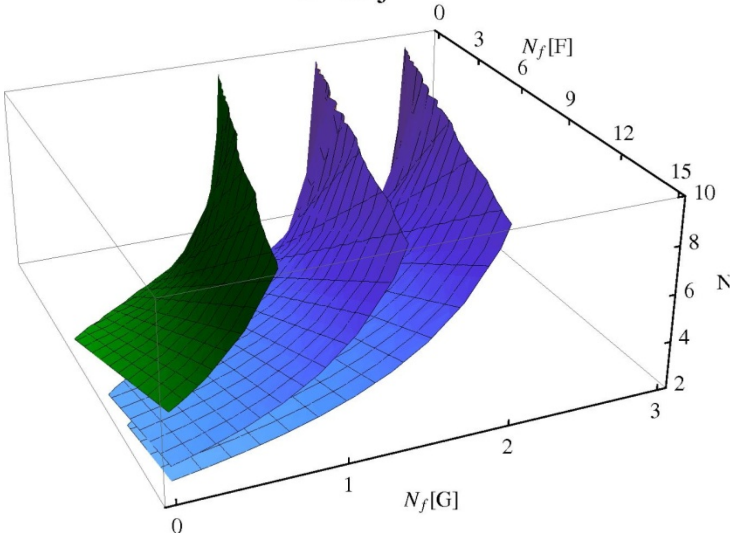
- Therefore the critical couplings calculated at two loops $\alpha_{c2}(r_1)$ and $\alpha_{c2}(r_2)$ will depend on the number of flavors and be gauge dependent.

- However: $\alpha_c(r_1) < \alpha_c(r_2) \Rightarrow \alpha_{c2}(r_1) < \alpha_{c2}(r_2)$ for all $N_f(r_i)$ and ξ

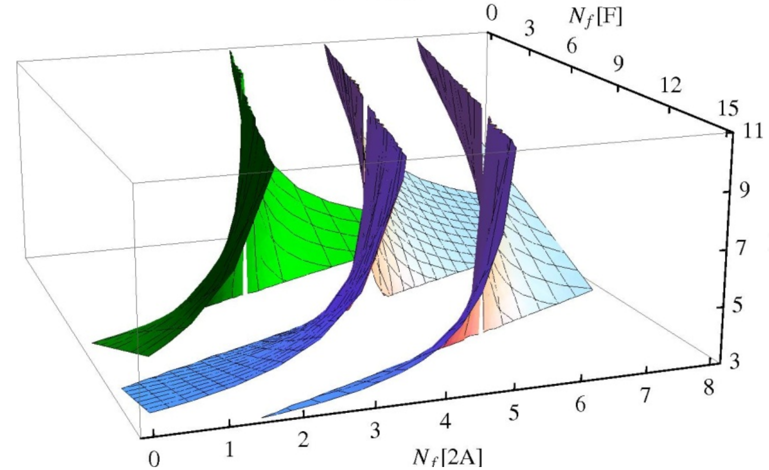
- Breaking pattern is not affected by higher orders.

- Assume: $C_2(r_1) > C_2(r_2)$ Conformal House: $\alpha_c(r_1) = \alpha_*(r_1, r_2)$

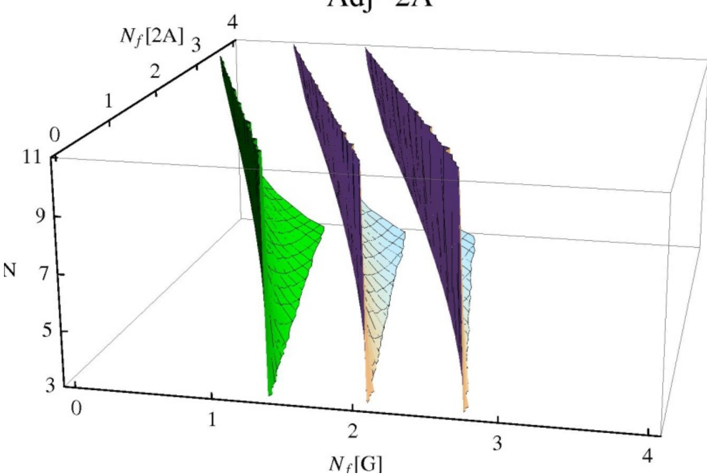
F−Adj



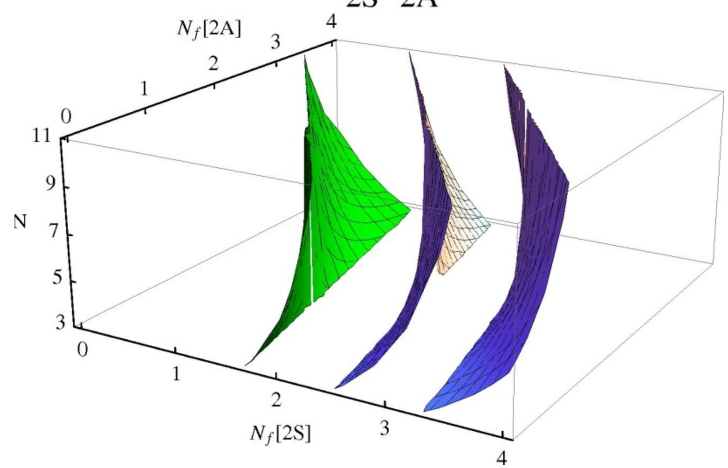
F−2A



Adj−2A



2S−2A



Part II

Technicolor and Beyond Standard Model Physics

Electroweak Symmetry Breaking

- Seek a dynamical origin of electroweak symmetry breaking.
- Several aspects to electroweak symmetry breaking:

1. Generate masses for W and Z bosons
2. Generate masses for the quarks and charged leptons
3. Why three generations?
4. Why do the masses exhibit the generational hierarchy that they do?

Technicolor

$$SU(N_{\mathrm{TC}}) \times SU(3) \times SU(2)_{\mathrm{L}} \times U(1)_{\mathrm{Y}}$$

- Electroweak symmetry breaks dynamically via the technicolor strong interactions

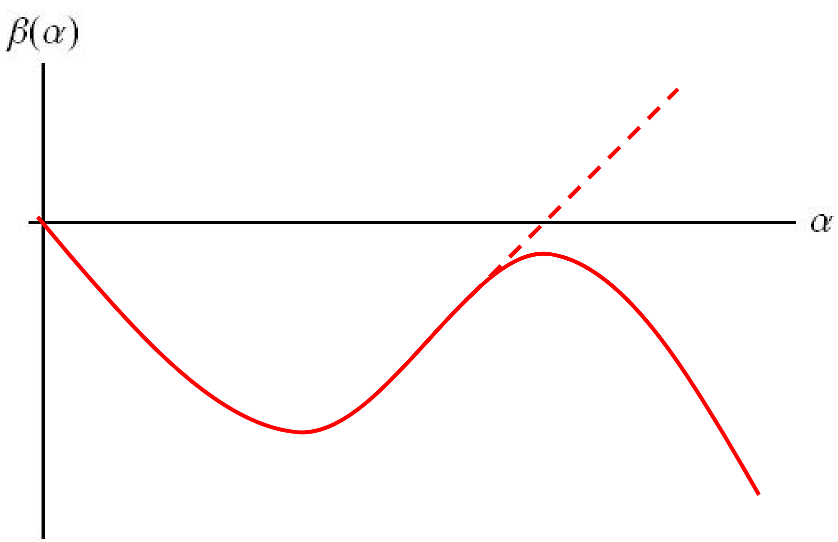
$$\left\langle Q^{c,f} \tilde{Q}_{c,f'} \right\rangle \neq 0 \qquad \Rightarrow \qquad \text{Electroweak symmetry breaks}$$

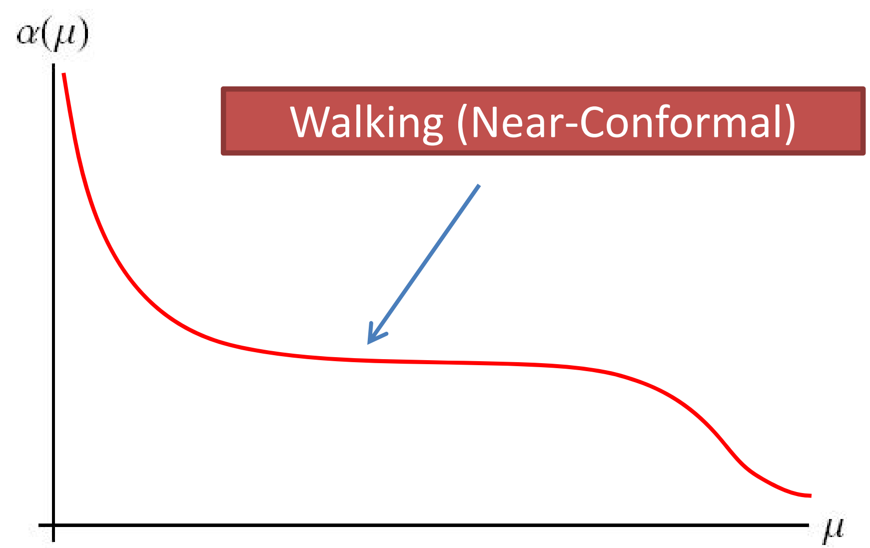
- W and Z bosons become massive.
- The Higgs is composite.

Weinberg, Susskind

Walking Technicolor

- Choose theory (colors, flavors, representations) just outside the conformal window





Holdom

Yamawaki, Bando, Matumoto

Chivukula, Cohen, Georgi

Minimal Walking Technicolor

- Naive S parameter

Peskin & Takeuchi

$$S_{\text{Naive}} = \frac{1}{6\pi}\frac{N_f}{2}d(r)$$

- Tension:

Walking dynamics ↔ Low S parameter

- S_{Naive} is an overestimation for walking theories.

Appelquist & Sannino

Kurachi & Shrock

- Phase diagram: Higher dimensional representations can save the day!
- Simplest theory:

$$SU(2) \text{ technicolor gauge group}$$
$$N_f = 2 \text{ in adjoint representation}$$

Sannino, Tuominen

Minimal Walking Technicolor

- Suffers from the Witten topological anomaly.

- Solution: Add new lepton family uncharged under technicolor interactions.

- Enhanced global symmetry

$$SU(4) \to SO(4)$$

- # Goldstone bosons = 15 – 6 = 9.

- Three are eaten.

Ultra Minimal Technicolor

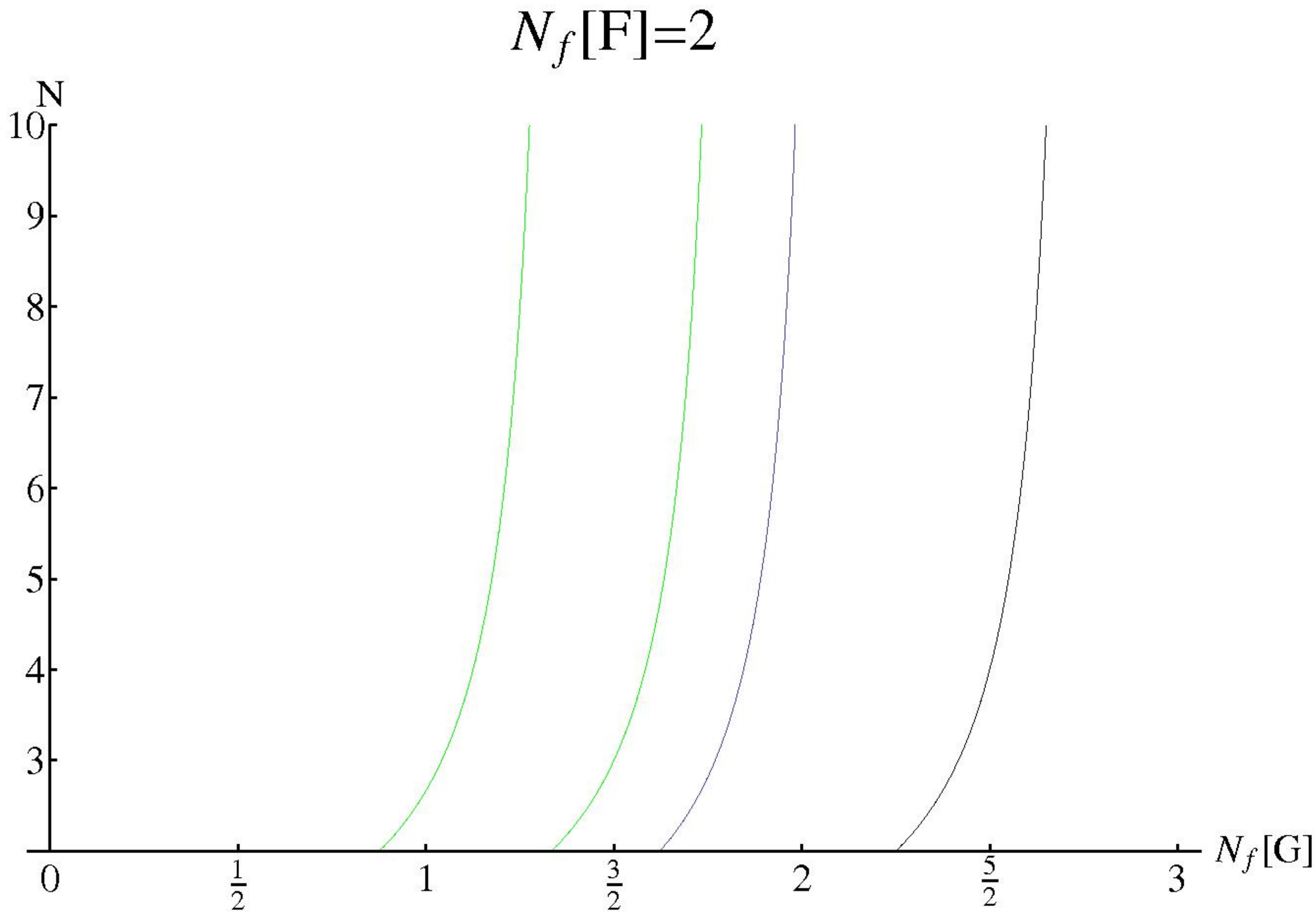
- Techniquarks in different representations of the gauge group. Eichten & Lane
- How do we pick our candidate theory? TR, Sannino
- 1st. criteria:

$$\text{Lowest naive S parameter} \quad \Rightarrow \quad \begin{cases} SU(2) \text{ gauge group} \\ N_f = 2 \text{ Dirac fermions in fundamental rep.} \end{cases}$$

- Problem: Theory is far from walking.
- One solution: Add remaining fundamental (5-6) flavors having no electroweak charges.

- Question: More economical solution?

  Answer: Higher dimensional representations.



- Our candidate theory

	$SU(2)_{TC}$	$SU(2)_L$	$U(1)_Y$
$(U_L, D_L)^T$	2	2	0
U_R	$\overline{2}$	1	½
D_R	$\overline{2}$	1	-½
λ^f	Adj	1	0

- Hypercharge is chosen such that the theory is anomaly free.

- The global symmetry breaks as follows

$$SU(4)\times SU(2)\times U(1)$$
$$\downarrow$$
$$Sp(4)\times SO(2)\times Z_2$$

- Leaving 15+3+1-10-1 = 8 Goldstone bosons.
- Both condensates break U(1) to the discrete Z_2 subgroup. Only λ^f is charged under Z_2

$$\lambda^f \to -\lambda^f, \qquad \text{under } Z_2.$$

Electroweak Symmetry Breaking

- Embed $SU(2)_L \times U(1)_Y$ in $SU(4)$.
- Once the (composite) Higgs develops a VEV the global symmetry breaks and the electroweak gauge group breaks to $U(1)_Q$.
- Three Goldstones are eaten by the electroweak gauge bosons.

	$U(1)_{TB}$	$U(1)_{T\lambda}$	Z_2
Π_{UD}	1	0	0
$\Pi_{\lambda\lambda}$	0	1	0
Θ	0	0	0

- All Goldstones are electroweak singlets and potentially DM candidates.

Extended Techicolor

- Seeking a dynamical origin of fermion masses.
- One gauges the generational indices and combines them with the Technicolor indices.
- The enlarged gauge symmetry is termed Extended Techicolor.
- The number of generations enters very differently than in the SM and MSSM.
- Since the dynamics highly depends on N_{gen} it is relevant to study ETC models taking N_{gen} as a variable. Is $N_{gen}=3$ special? TR, Shrock

Extended Techicolor

- Gauge group $SU(N_{ETC}) \times SU(N_{HC}) \times SU(3) \times SU(2)_L \times U(1)_Y$.

- $SU(N_{ETC})$ is chiral and self-breaks in a series of stages to $SU(N_{TC})$. (It tumbles).

- $N_{ETC} = N_{gen} + N_{TC}$

- Take simplest case of two technicolors: $N_{ETC} = N_{gen} + 2$.
- Also take $N_{HC}=2$.

- Fermion content (leaving out hypercharge)

$Q_L : (N_{ETC},1,3,2)_L$ $\quad$ $u_R : (N_{ETC},1,3,1)_R$ $\quad$ $d_R : (N_{ETC},1,3,1)_R$

$L_L : (N_{ETC},1,1,2)_L$ $\quad$ $e_R : (N_{ETC},1,1,1)_R$

- The theory is chiral and we need to add additional SM-singlet matter in order to be free of gauge anomalies.

- The self-breaking should be of the form

$SU(N_{ETC})$	$\rightarrow$	$SU(N_{ETC}-1)$	at Λ_1
	:		
	:		
	:		
$SU(3)$	$\rightarrow$	$SU(2)$	at $\Lambda_{n_{gen}}$

Where $\Lambda_1 > \Lambda_2 > \ldots. > \Lambda_{n_{gen}}$.

- One should be careful about the highly attractive channel

$N_{ETC} \times \overline{N}_{ETC} \rightarrow 1$ with measure of att: $\Delta C_2 = \frac{N_{ETC}^2 - 1}{N_{ETC}}$

$N_{gen} = 1$

- Gauge group $SU(3)_{ETC}\times SU(2)_{HC}\times SU(3)\times SU(2)_L\times U(1)_Y$
- The theory is anomaly free if we add

$$\left(\bar{3},1,1,1\right)_R \qquad \left(3,2,1,1\right)_R \qquad \left(1,2,1,1\right)_R$$

- The theory depends on the relative strenghts of $SU(3)_{ETC}$ and $SU(2)_{HC}$.
- Choose hyper color to be dominant. Then the preferred channel is

$$\left(3,2,1,1\right)_R\times\left(3,2,1,1\right)_R\rightarrow\left(\bar{3},1,1,1\right)_R \text{ at } \Lambda_1, \quad \text{breaking} \quad SU(3)_{ETC}\rightarrow SU(2)_{TC}$$

- Below Λ_1 we are left with $\left(2,1,1,1\right)_R \quad \left(1,2,1,1\right)_R \quad \left(1,2,1,1\right)_R$

$$\left(1,2,1,1\right)_R\times\left(1,2,1,1\right)_R\rightarrow\left(1,1,1,1\right)_R \quad \text{at} \quad \Lambda_1^{'}<\Lambda_1$$

- Finally left with 7.5 + 0.5 = 8 Dirac fermions.

- Hyper color insures condensation in the right channel.

- The condensate breaks $SU(3)_{ETC}$ → $SU(2)_{TC}$ and gives mass to the one generation of standard model fermions.

- The remaining Technicolor theory walks (8 Dirac fermions) and breaks electroweak symmetry.

- Fermion masses

$$m_f \simeq \kappa\eta \frac{\Lambda_{TC}^3}{\Lambda_1^2}, \qquad \eta = \exp\left[\int_{\Lambda_{TC}}^{\Lambda_1} \frac{d\mu}{\mu}\gamma\left(\alpha\left(\mu\right)\right)\right], \qquad k \sim O(10)$$

- Walking: $m_f \simeq \kappa \frac{\Lambda_{TC}^2}{\Lambda_1}$, Non-walking: $m_f \simeq \kappa \frac{\Lambda_{TC}^3}{\Lambda_1^2}$,

Warning 1: A variant of the $N_{gen} = 1$ Model

- The theory is anomaly free if we add:

$$3\left(3,1,1,1\right)_R \qquad \left(\bar{3},2,1,1\right)_R \qquad \left(1,2,1,1\right)_R$$

- MAC:

$$\left(\bar{3},2,1,1\right)_R \times \left(\bar{3},2,1,1\right)_R \to \left(3,1,1,1\right)_R$$ At Λ_1, breaking $SU(3)_{ETC} \to SU(2)_{TC}$

- Below Λ_1 we are left with $3\left(2,1,1,1\right)_R \quad \left(1,2,1,1\right)_R \quad \left(1,2,1,1\right)_R$

$$\left(1,2,1,1\right)_R \times \left(1,2,1,1\right)_R \to \left(1,1,1,1\right)_R$$ At $\Lambda_1^{'} < \Lambda_1$

- Warning: Below Λ_1' we are left with 7.5 + 1.5 = 9 Dirac fermions. Therefore the Technicolor theory is conformal.

Warning 2: A variant of the $N_{gen} = 1$ Model. Only ETC symmetries

- The theory is anomaly free if we add:

$$6\left(\bar{3},1,1,1\right)_R \qquad \left(6,1,1,1\right)_R$$

- MAC

$$\left(\bar{3},1,1,1\right)_R \times \left(6,1,1,1\right)_R \rightarrow \left(3,1,1,1\right)_R$$

- However: $\alpha_{ETC,IR} \simeq 0.32$ $\qquad \alpha_{cr} \simeq 0.63$

- Warning: The ETC theory is conformal.

General Considerations

Appelquist, Christensen, Piai, Shrock, TR

- The picture is the same for N_{gen} = 1,2,3,4.
- Hyper color group is needed in all cases.
- When adding the additional fermions one should check that the ETC theory or the resultant TC theory is not IR-conformal.
- It does not seem to become more complicated as one increases the number of generations.
- What additional information is needed to show that N_{gen} = 3 is speciel?

Conclusions

- Investigated phase diagram using the ladder approximation and conjectured all-orders beta function: The conformal window and the conformal house.

- Is the all-orders beta function exact? Dualites?

- Constructed two explicit TC model: MWT and UMT.
- Investigated the generational structure in a class of ETC models.
- Still lots to do……

The End