Monopoles, bions, and other oddballs in confinement or conformality

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with Mithat Ünsal — SLAC/Stanford

things one would like to understand about any theory:

- does it confine?
- does it break its (super) symmetries?
- is it conformal?
- what are the spectrum, interactions...?

tough to address, in almost all theories
but interesting for:

satisfying theoretical curiosity
QCD
SUSY extensions of the Standard Model
non-SUSY extensions of the Standard Model

conventional wisdom:

SUSY - very "friendly"
beautiful - exact results

pure YM - formal `but see www.claymath.org/millennium/`

QCD-like - hard, leave it to lattice folks

`but recall (a,V,m,$)`

non-SUSY chiral gauge theories - even lattice not practical
...almost nobody talks about them anymore

what I'll talk about applies to any of the above theories

have we solved these theories?

...

we study a regime where the nonperturbative dynamics of 4d gauge theories

- SUSY or not, chiral or vectorlike -

is analytically tractable:

compactifying 4d gauge theories on a small circle is a "deformation" where nonperturbative dynamics is under theoretical control

- dynamics as "friendly" as SUSY, e.g. Seiberg-Witten theory in SUSY theories, "circle deformation" was pursued in late '90s: then "forgotten", even within SUSY theory space

Seiberg, Witten (N=2 SYM)
Aharony, Hanany, Intriligator, Seiberg, Strassler;
Davies, Hollowood, Khoze, Mattis (N=1 SYM/SQCD)

a revival has occurred recently - both in SUSY and non-SUSY

Unsal; Unsal, Yaffe; Unsal, Shifman; Unsal, EP (2007-2009)

punchline:

we gain new, sometimes (perhaps) surprising, insight into the physics of confinement and abelian or discrete chiral symmetry breaking in vectorlike and chiral gauge theories with massless fermions
- all in a "locally 4d" setting

however: "friendliness" on $R^3 \times S^1$ does not extend to R^4
... except very few special cases, not all SUSY

do not expect to compute detailed properties of QCD, or other theories

- some qualitative information on the phase structure of the theories is likely to be relevant, however
- at the end, we will attempt to "estimate" the critical number of massless fermion species where theory becomes conformal
- perhaps surprisingly, we will see that results of very different uncontrolled calculations agree reasonably well with each other & with "experiment" (i.e. lattice, whenever available)

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I should stress: compactifying 4d theory on a small circle is no magic bullet.

Many aspects of the physics are not accessible in this regime - but the relative simplicity is what gives us theoretical control!

It appears, however, that at least some aspects of the dynamics relevant for the transition to conformality are retained. **Especially interesting in theories with confinement without chiral symmetry breaking - as in SUSY or perhaps some non-SUSY (chiral?) theories where lattice methods fail.**

The plan

of this talk is to tell you, largely in pictures, what the above statements amount to.

time ↑ inf{rigour} ↓

Conformality or confinement (II): One-flavor CFTs and mixed-representation QCD
JHEP 0912:011,2009; 0910.1245, 33pp

Conformality or confinement: (IR)relevance of topological excitations
JHEP 0909:050,2009; 0906.5156, 42pp

Chiral gauge dynamics and dynamical supersymmetry breaking
JHEP 0907:060,2009; 0905.0634, 31pp

Index theorem for topological excitations on R*3 x S*1 and Chern-Simons theory
JHEP 0903:027,2009; 0812.2085, 29pp

(all by M. Unsal and E.P.)

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3d Polyakov model & "monopole-instanton"-induced confinement — Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$ — K. Lee, P. Yi, 1997; P. van Baal, 1998

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center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations — Shifman, Unsal, 2008; Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

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Polyakov, 1977

continuum picture: 3d Georgi-Glashow [on the lattice - compact U(1)]

$$L \sim \frac{1}{g_3^2}\left(F^a_{\mu\nu} F^a_{\mu\nu} + D_\mu \phi^a D^\mu \phi^a\right) \qquad \mu,\nu = 1,2,3$$

$$[A_\mu] = [\phi] = 1 \qquad [g_3^2] = 1$$

due to some Higgs potential $\langle \phi \rangle = (0, 0, v)$

$SU(2) \xrightarrow{v} U(1)$ at low energies, $E \ll m_W \sim v$

free U(1) theory $A^3_\mu \equiv A_\mu$

$$\mathcal{L}_{eff} = \frac{1}{g_3^2} F_{\mu\nu}^2 + \ldots$$

"..." are perturbatively calculable & not very interesting

$B_\mu = \epsilon_{\mu\nu\lambda} F_{\nu\lambda}$ "magnetic field"
topologically conserved current of **"emergent topological U(1) symmetry"** responsible for conservation of magnetic charge

$B_\mu = g_3^2 \, \partial_\mu \sigma$ 3d photon dual to scalar (as one polarization only)

$\partial_\mu B_\mu = 0$ Abelian duality $\partial_\mu^2 \sigma = 0$

Bianchi identity $\longleftrightarrow$ equation of motion

$\mathcal{L}_{eff} = \frac{1}{g_3^2} F_{\mu\nu}^2 + \ldots$ $\mathcal{L}_{eff} = g_3^2 (\partial_\mu \sigma)^2 + \ldots$

topological U(1) symmetry = shift of "dual photon"

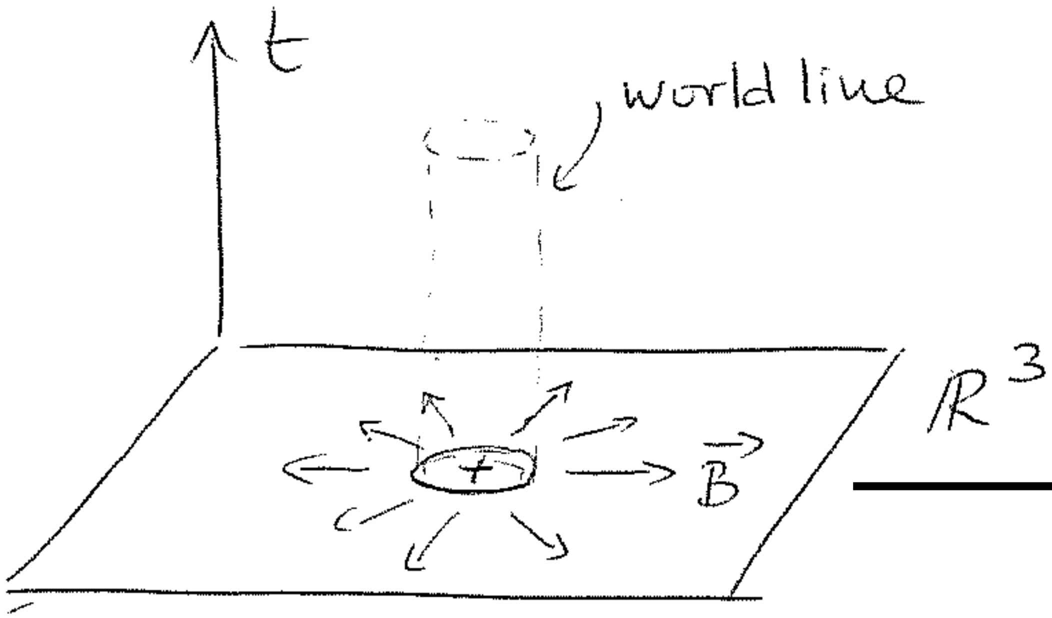
a rather ***"boring-boring" duality* -** if not for the existence of monopoles:

monopoles $\partial_\mu B_\mu$ = quantized magnetic charge - shift symmetry broken

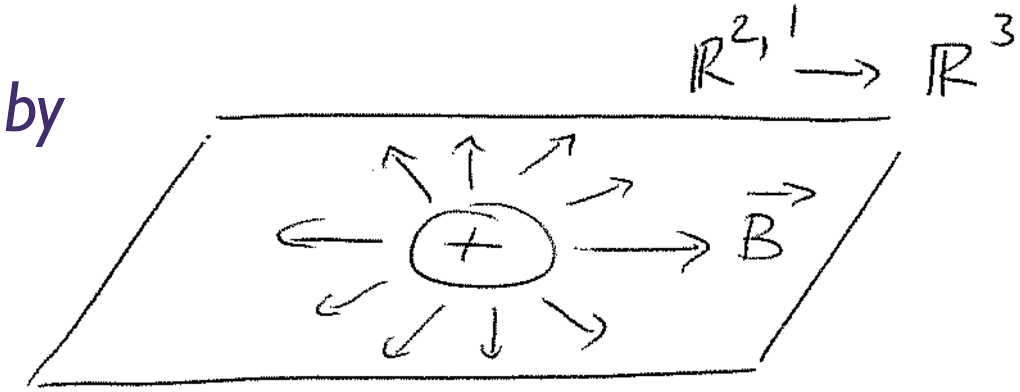
- dual photon gains mass & **electric charges confined**

how?
...in pictures:

"'t Hooft-Polyakov monopole" - *static finite energy solution of Georgi-Glashow model in 4d*



get Euclidean 3d by "forgetting time"



solution of Euclidean eqns. of motion of finite action: a "monopole-instanton"

$$E_M = \frac{4\pi v}{g_4^2} \longrightarrow S_0 = \frac{4\pi v}{g_3^2}$$

$$e^{-S_0} \to 0 \quad \text{as } g_3^2/v \to 0$$

M-M* pairs give exponentially suppressed (at weak coupling) "semiclassical" contributions to the vacuum functional

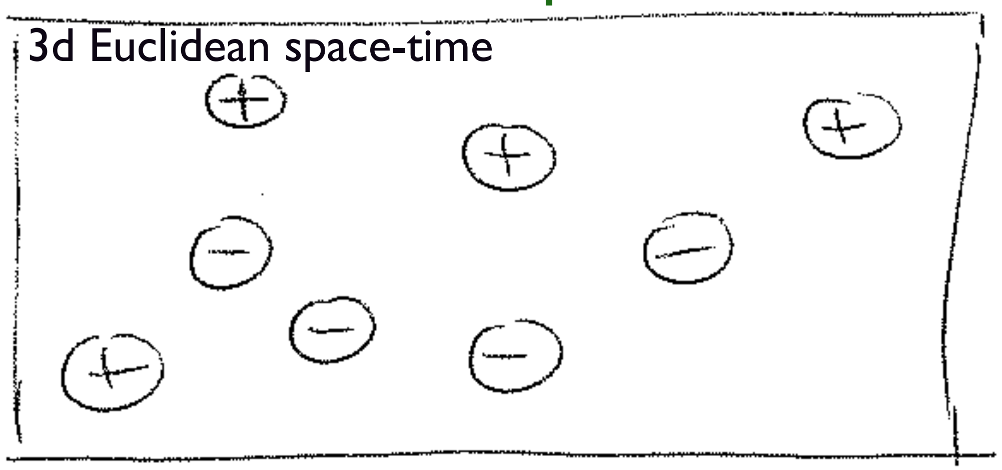
vacuum "is" a dilute monopole-antimonopole plasma

number of M's per unit volume ~ $v^3 e^{-S_0}$

(analogous to B+L violation in electroweak model; at T=0 exponentially small)

vacuum is a dilute M-M* plasma - but interacting, unlike instanton gas in 4d (in, say, electroweak theory)

in pictures & in formulae



$Z =$
Z(perturbative)x
Z(charged plasma with Coulomb interactions)

really meaning grand partition function of classical 3d M-M* plasma

physics is that of Debye screening - analogy:

electric fields are screened in a charged plasma ("Debye mass for photon"), so in the monopole-antimonopole plasma, the dual photon obtains mass from screening of magnetic field:

$$\mathcal{L}_{eff} = g_3^2 (\partial\sigma)^2 + (\#)\, v^3 e^{-S_0}\left(e^{i\sigma} + e^{-i\sigma}\right) + \ldots$$

"(anti-)monopole operators"

aka **"disorder operators"** - not locally expressed in terms of original gauge fields (Kadanoff-Ceva; 't Hooft - 1970s)

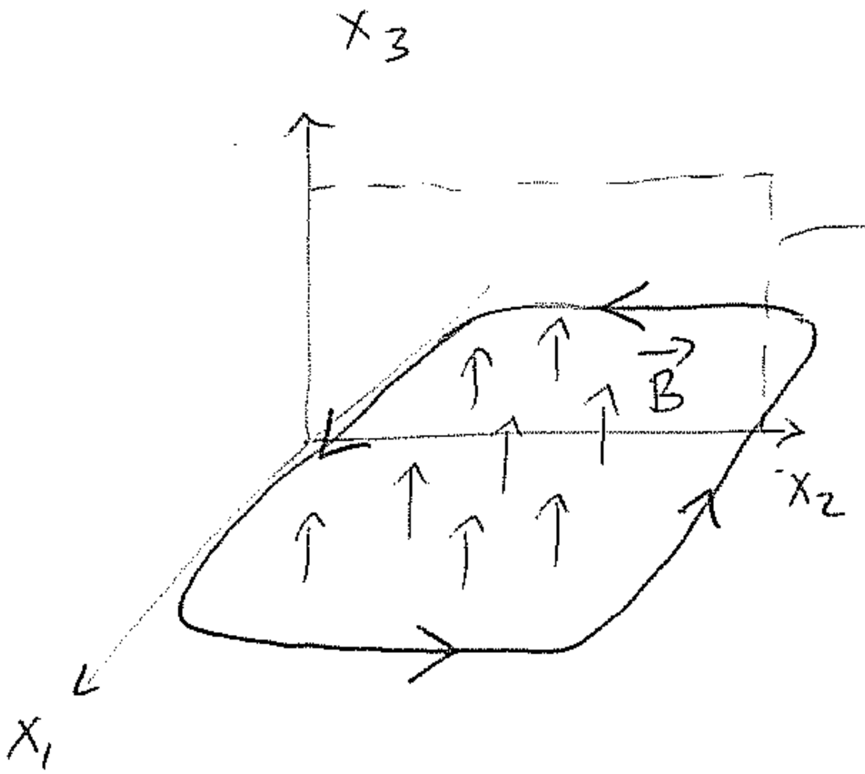
also by analogy with Debye mass:

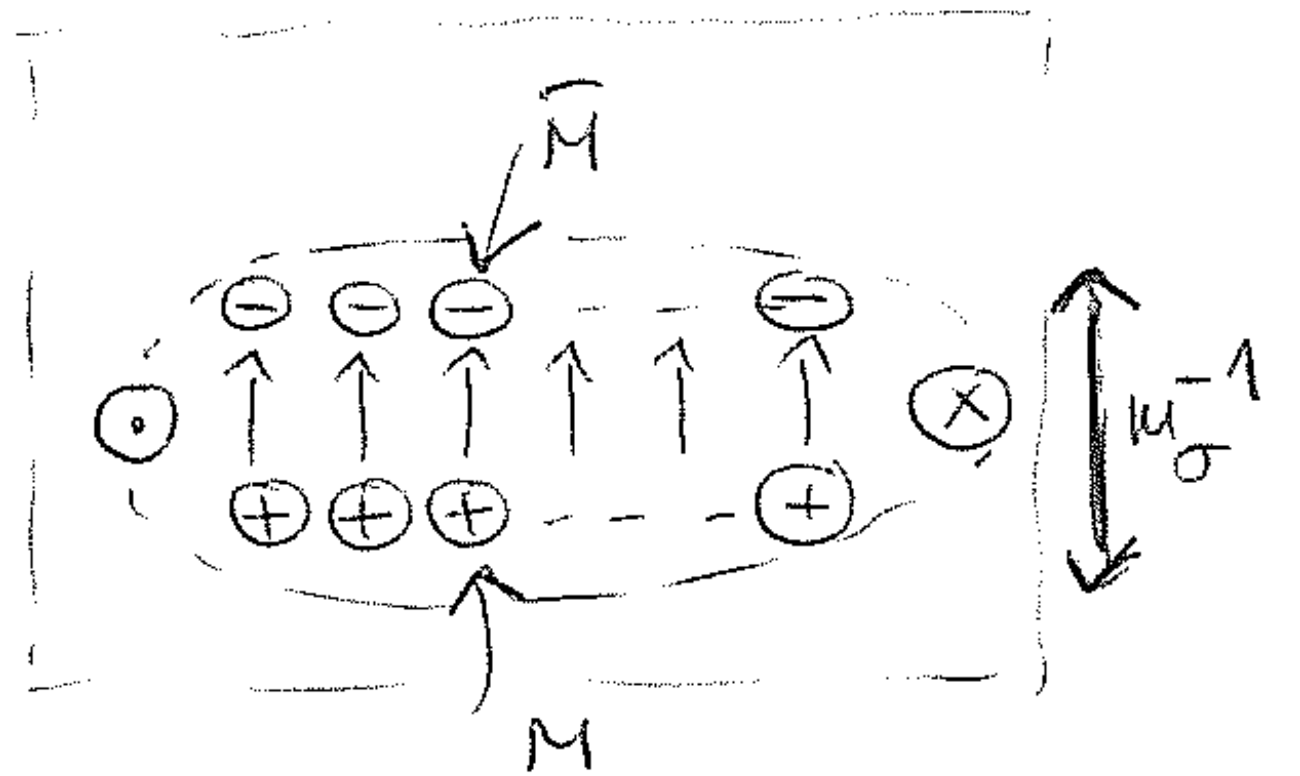
dual photon mass^2 ~ M-M* plasma density

$$m_\sigma \sim v\, e^{-S_0/2} = v\, e^{-\frac{4\pi v}{2 g_3^2}}$$

next:
dual photon mass
~ confining string tension...

in pictures:

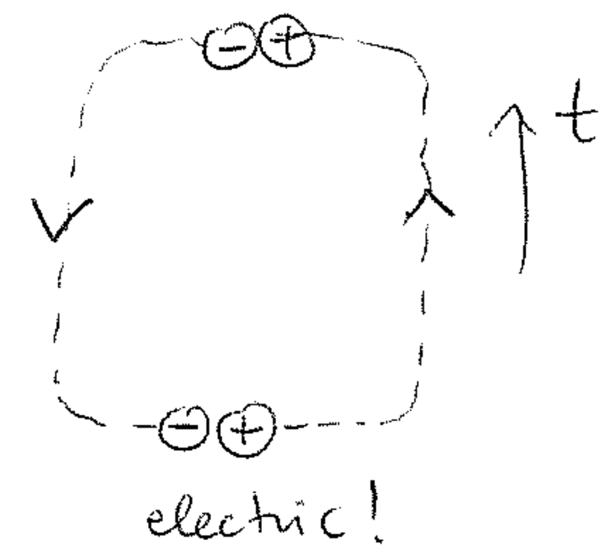




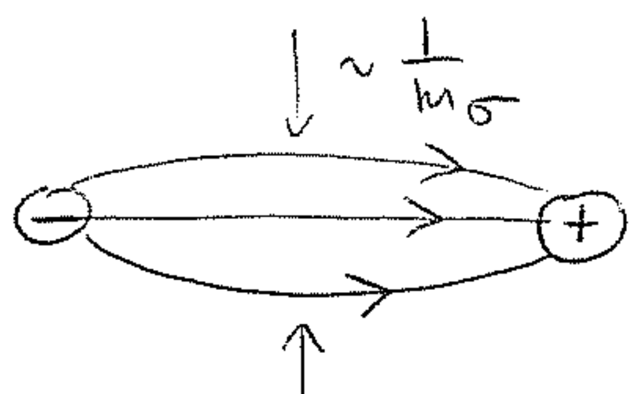
screening of magnetic field in plasma
= Wilson loop area law:

$$\langle e^{i\oint A\,dx} \rangle \sim e^{-(\text{Area})\, m_\sigma g_3^2}$$

Minkowski space interpretation of Wilson loop: x_1 - time



electric flux tube



confining flux tube: **tension^{-1} ~ thickness ~ inverse dual photon mass**

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"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

we want to go to 4d - by "growing" a compact dimension: $S^1: \quad x^4 \sim x^4 + L$

"monopole-instantons" on $R^3 \times S^1$	K. Lee, P. Yi, 1997 P. van Baal, 1998

A_4 is now an adjoint 3d scalar Higgs field $\quad \partial_4 + A_4 \longrightarrow \frac{2\pi n}{L} + A_4$

but it is a bit unusual -
a compact Higgs field: $\quad \langle A_4 \rangle \sim \langle A_4 \rangle + \frac{2\pi}{L}$ such shifts of A_4 vev absorbed into shift of KK number "n"

thus, natural scale of "Higgs vev" is $\quad \langle A_4 \rangle \sim \frac{\pi}{L}$ leading to $SU(2) \xrightarrow{\frac{1}{L}} U(1)$

(clearly, semiclassical and weakly coupled if L << strong scale)

$\langle A_4 \rangle \sim \frac{\pi}{L}$ or, equivalently, Wilson line vev: $\langle W \rangle = \begin{pmatrix} e^{i\pi/2} & \\ & e^{-i\pi/2} \end{pmatrix}$

breaks SU(2) to U(1) so there are monopoles:

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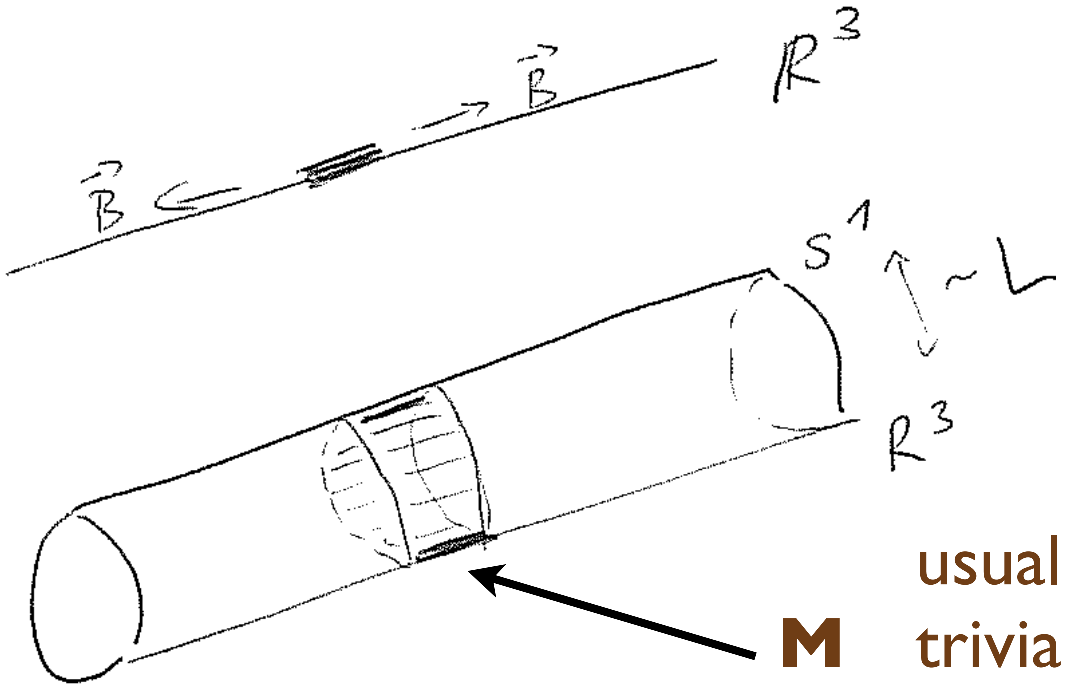

usual monopole
trivially
embedded in 4d

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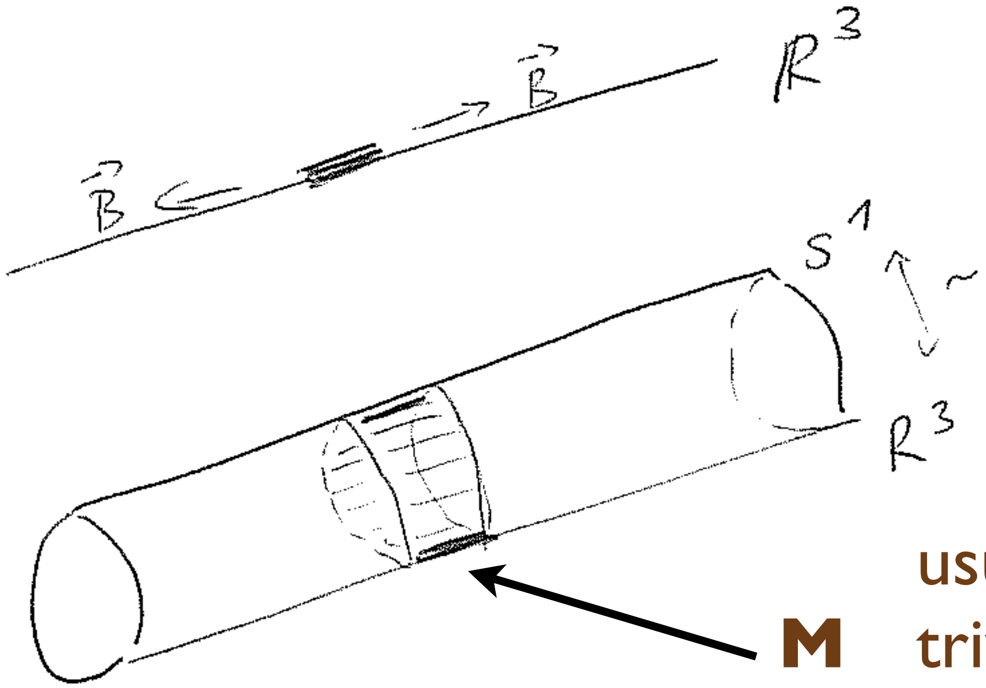
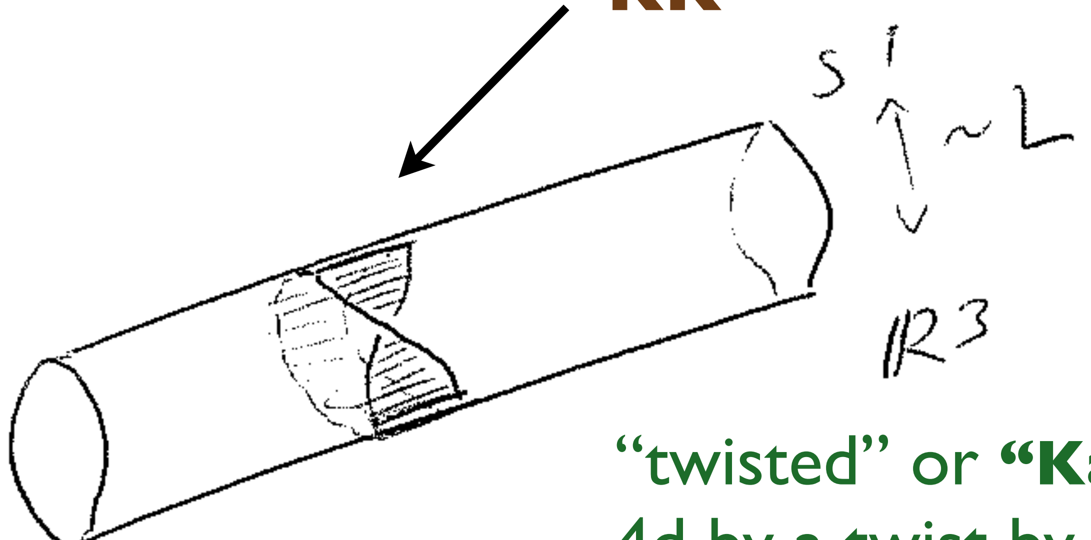


M usual monopole trivially embedded in 4d

KK



"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)

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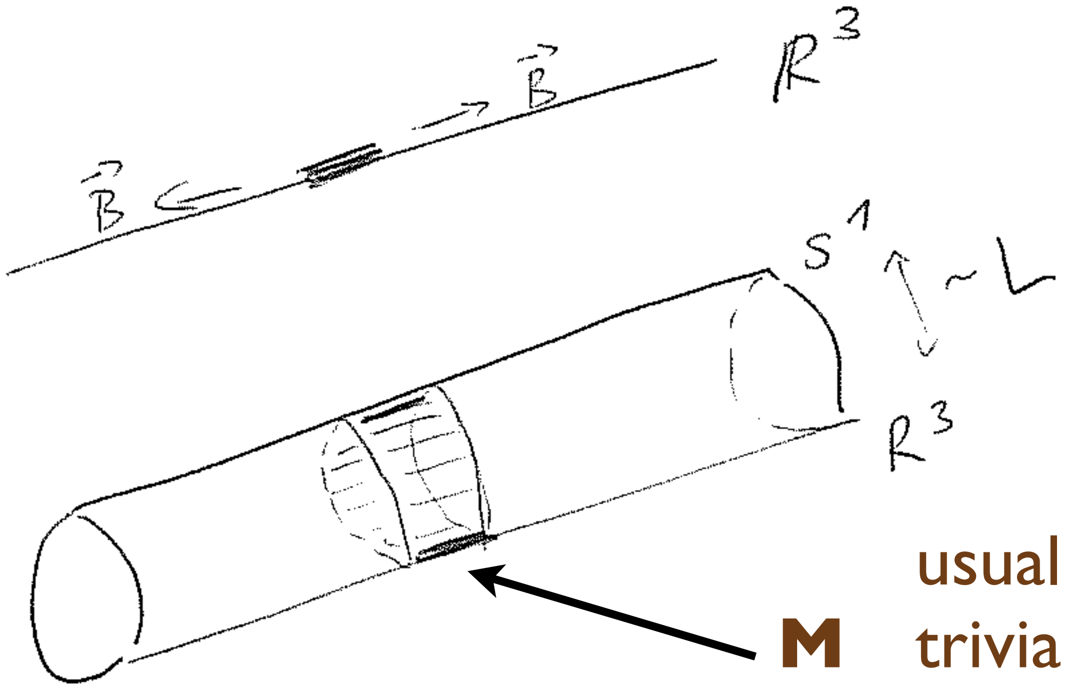
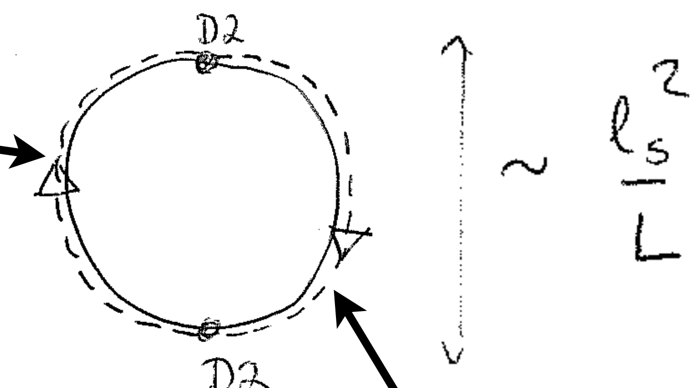


M

usual monopole trivially embedded in 4d

KK discovered by K. Lee, P. Yi, 1997, as *"Instantons and monopoles on partially compactified D-branes"*

M
Euclidean
D0-brane



KK
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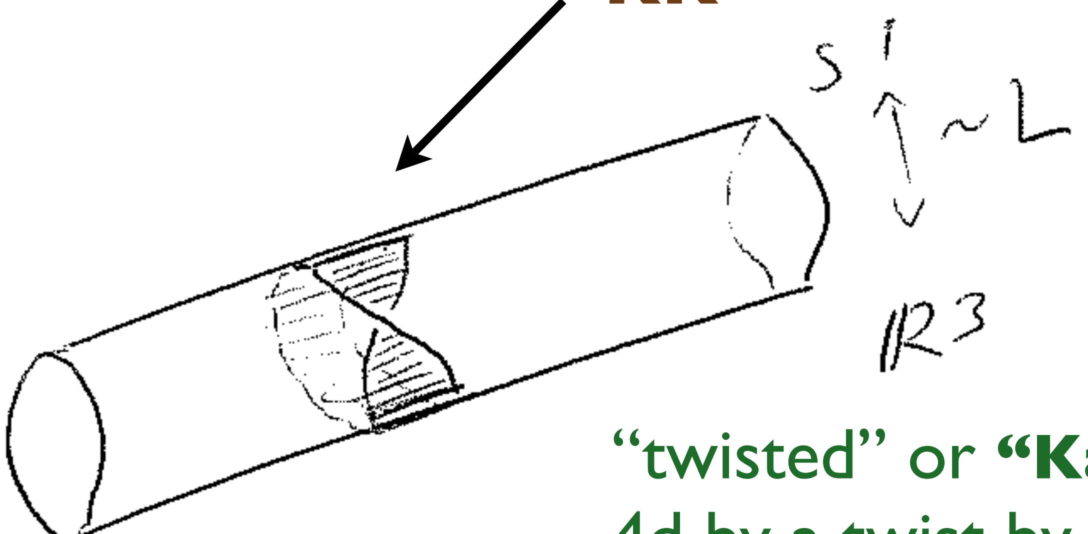
KK



"twisted" or **"Kaluza-Klein":** monopole embedded in 4d by a twist by a "gauge transformation" periodic up to center - in 3d limit not there! (infinite action)

D-brane picture is very useful, but perhaps "KK monopoles" not so new...

X is the "Higgs field"

V are not well defined. Then V has a line of directional singularities, which one can interpret as the world line of a magnetic monopole (in euclidean space). In the generic case the world lines intersect the three dimensional region Ω in a discrete set of points. Because the eigenvalues of X are ordered, only adjacent pairs can become degenerate. If $\lambda_i = \lambda_{i+1}$, we shall label such a point $x^{(i)}$. Should X be an element of the group SU(N), one must keep in mind that (2.9) also admits $\lambda_1 = \lambda_N$ with $\phi_1 = \phi_N + 2\pi$. We shall label such points $x^{(0)}$ or $x^{(N)}$. Away from all $x^{(i)}$ the currents vanish because V is then differentiable often enough. It is therefore

Nuclear Physics B293 (1987) 461–478
North-Holland, Amsterdam

TOPOLOGY AND DYNAMICS OF THE CONFINEMENT MECHANISM

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Summary: "elementary" topological excitations on R3xS1

M & KK both self-dual objects, of opposite magnetic charges

	magnetic charge	topological charge	suppression
M	+1	1/2	e^{-S_0}
KK	-1	1/2	e^{-S_0}
BPST	0	1	e^{-2S_0}

+ their anti-"particles"

- thus, BPST instanton " = M+KK " (aka "calorons" P. van Baal, 1998)

$$e^{-S_0} = e^{-\frac{4\pi v}{g_3^2}} = e^{-\frac{4\pi^2}{L g_3^2}} = e^{-\frac{4\pi^2}{g_4^2(L)}}$$

$$SU(N): \quad e^{-S_0} = e^{-\frac{8\pi^2}{g_4^2(L) N}}$$

(large-N survive!)

M & KK have, in SU(N), 1/N-th of the 't Hooft suppression factor aka: "fractional instantons", "instanton quarks", "zindons", "quinks", "instanton partons"... [collected by D. Tong]

Next, to understand the role M, KK, M* & KK* play in various theories of interest, need to know what happens to the operators they induce when there are fermions in the theory.

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"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

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- for some theories the answer for the number of zero modes in M or KK background had been guessed (correctly)

  e.g. SUSY YM - Aharony, Hanany, Intriligator, Seiberg, Strassler, 1997

- while studying ISS(henker) proposal for SUSY breaking model, Unsal and I needed a general index theorem [SU(2)+three-index symm. tensor Weyl]

- we found this:

Journal of Functional Analysis **177**, 203–218 (2000)
doi:10.1006/jfan.2000.3648, available online at http://www.idealibrary.com on IDEAL®

An L^2-Index Theorem for Dirac Operators on $S^1 \times \mathbb{R}^3$

Tom M. W. Nye and Michael A. Singer

where, in APPENDIX A. ADIABATIC LIMITS OF η-INVARIANTS

we found:

$$\text{ind } (D^+_{\mathbb{A}}) = \int_X \text{ch}(\mathbb{E}) + \frac{1}{\mu_0} \sum_{\mu} \epsilon_\mu c_1(E_\mu)[S^2_\infty]$$

$$= \int_X \text{ch}(\mathbb{E}) - \frac{1}{2}\overline{\eta}_{\text{lim}}$$

(last formula in paper)

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(last formula in paper)

two obvious questions:

1.) where does this come from?

2.) what number is it equal to in a given topological background (M,KK...)
& how does it depend on ratio of radius to holonomy?

for answers & more

see M. Unsal, EP 0812.2085

like on R^3 Callias ← physicist derivation — E. Weinberg, 1970s, but on $R^3 \times S^1$, so must incorporate anomaly equation, some interesting effects

for this talk it is enough to consider 4d SU(2) theories with N_w adjoint Weyl fermions

"applications": $N_w=1$ is N=1 SUSY YM

$N_w=4$
- "minimal walking technicolor"
- happens to be N=4 SYM without the scalars (a gravity dual, anyone?)

M KK M* KK* each have $2N_w$ zero modes

disorder operators:

M: $e^{-S_0}\, e^{i\sigma}\, (\lambda\lambda)^{N_w}$

KK: $e^{-S_0}\, e^{-i\sigma}\, (\lambda\lambda)^{N_w}$

M*: $e^{-S_0}\, e^{-i\sigma}\, (\bar\lambda\bar\lambda)^{N_w}$

KK*: $e^{-S_0}\, e^{i\sigma}\, (\bar\lambda\bar\lambda)^{N_w}$

where:

$$(\lambda\lambda)^{N_w} = \det_{I,J} \lambda^a_{\alpha I}\, \lambda^a_{\beta J}\, \epsilon^{\alpha\beta}$$

(I,J: $SU(N_w)$; a: $SU(2)$; α,β: $SL(2,\mathbb{C})$)

remarks:

- operator due to M+KK = 't Hooft vertex; independent of dual photon
- "our" index theorem interpolates between 3d Callias and 4d APS index thms.

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"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

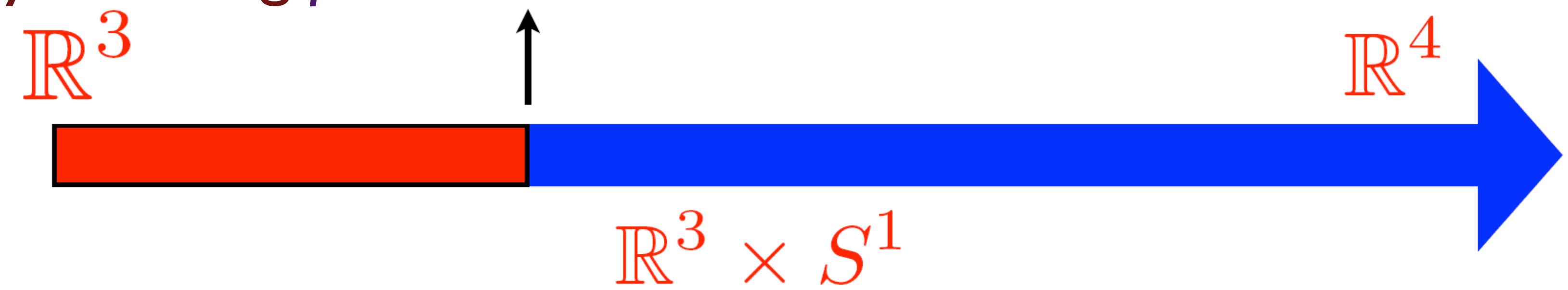
- Abelianization occurs only if there is a nontrivial holonomy (i.e., A_4 has vev)

- upon thermal circle compactifications, gauge theories with fermions do not Abelianize: center symmetry is broken at small circle size - transition to a deconfining phase - $A_4 = 0$, $\langle trW \rangle \neq 0$ - deconfinement - at high-T, 1-loop V_{eff}
  Gross, Pisarski, Yaffe, early 1980s

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations

Shifman, Unsal, 2008
Unsal, Yaffe, 2008

in other words, in thermal setup, upon decompactification, we have a center-symmetry breaking *phase transition* and no smooth connection to R^4



to ensure calculability at small L and smooth connection to large L in the sense of center symmetry: *can one find ways to avoid phase transition?*

I. non-thermal compactifications - periodic fermions ("twisted partition function")

- with $N_W > 1$ adjoint fermions center symmetry preserved (Unsal, Yaffe 2007) as well as with other, "exotic" fermion reps (Unsal, EP 2009)
- in many supersymmetric theories, can simply choose center-symmetric vev

II. add double-trace deformations: force center symmetric vacuum at small L (Shifman, Unsal 2008)

In what follows, we assume center-symmetric vacuum - due to either I. or II. - will explicitly discuss only theory where center symmetry is naturally preserved at small L (**I**.)

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"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

First, the key players:

ready to study the dynamics of theories with massless fermions on a small circle

in a vacuum with A_4 vev, abelianization:

- in SU(2): (dual) photon massless + fermion components w/out mass from vev (neutral)

- monopoles + KK monopoles are the basic topological excitations

is there magnetic field screening in the vacuum?

the answer would appear to be "no": since M and KK have fermion zero modes, monopole operators do not generate potential for dual photon,
so, **no screening & no confinement... ?**

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement

Unsal, 2007
Unsal, EP, 2009

but take a look at the symmetries first:

as an example, again consider 4d SU(2) theories with N_w adjoint Weyl fermions

classical global chiral symmetry is

$$SU(N_w) \times U(1)$$

but 't Hooft vertex $(\lambda\lambda)^{2N_w} e^{-\frac{8\pi^2}{g_4^2}}$ only preserves $\mathbb{Z}_{4N_w}: \lambda \to e^{i\frac{2\pi}{4N_w}} \lambda$

so, quantum-mechanically we have only $SU(N_w) \times Z_{4N_w}$ exact chiral symmetry

now **M, KK**(+*) operators all look like: $e^{-S_0} e^{i\sigma} (\lambda\lambda)^{N_w}$

hence $(\lambda\lambda)^{N_w} \xrightarrow{\mathbb{Z}_{4N_w}} e^{i\pi} (\lambda\lambda)^{N_w}$

invariance of **M**, **KK**(+*) operators under exact chiral symmetry means that

dual photon must transform under the exact chiral symmetry

i.e., topological shift symmetry is intertwined with chiral symmetry:

$$\mathbb{Z}_{4N_w}: \quad \sigma \to \sigma + \pi$$

$\sigma \to \sigma + \pi$ ~~$\cos\sigma$~~ $\cos(2\sigma)$ ✓

so the exact chiral symmetry allows a potential - **but what is it due to?**

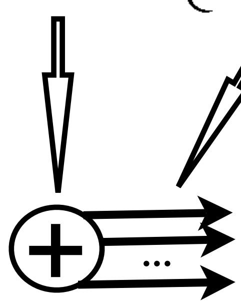
to generate $\cos(2\sigma)$ must have

i. magnetic charge 2

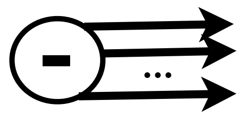
ii. no zero modes

$$e^{-S_0}\, e^{i\sigma}\, (\lambda\lambda)^{N_w}$$

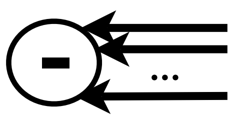
M:



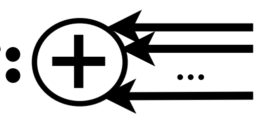
KK:



M*:



KK*:

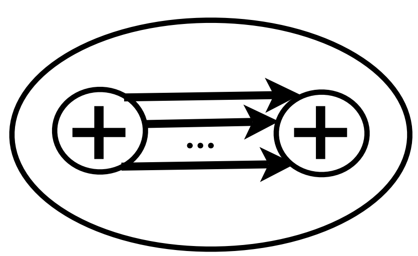


M + KK* bound state? (Unsal, 2007)

- same magnetic charge ~ 1/r-repulsion
- fermion exchange ~ log(r)-attraction

M + KK* = B - magnetic "bion" **(size of bion is ~ $1/g_3^2 \gg 1/v$)**

B:



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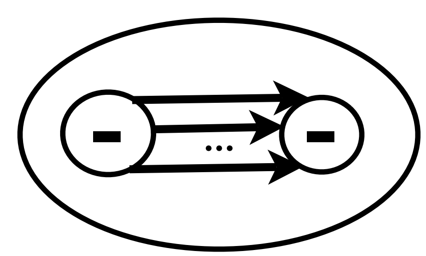
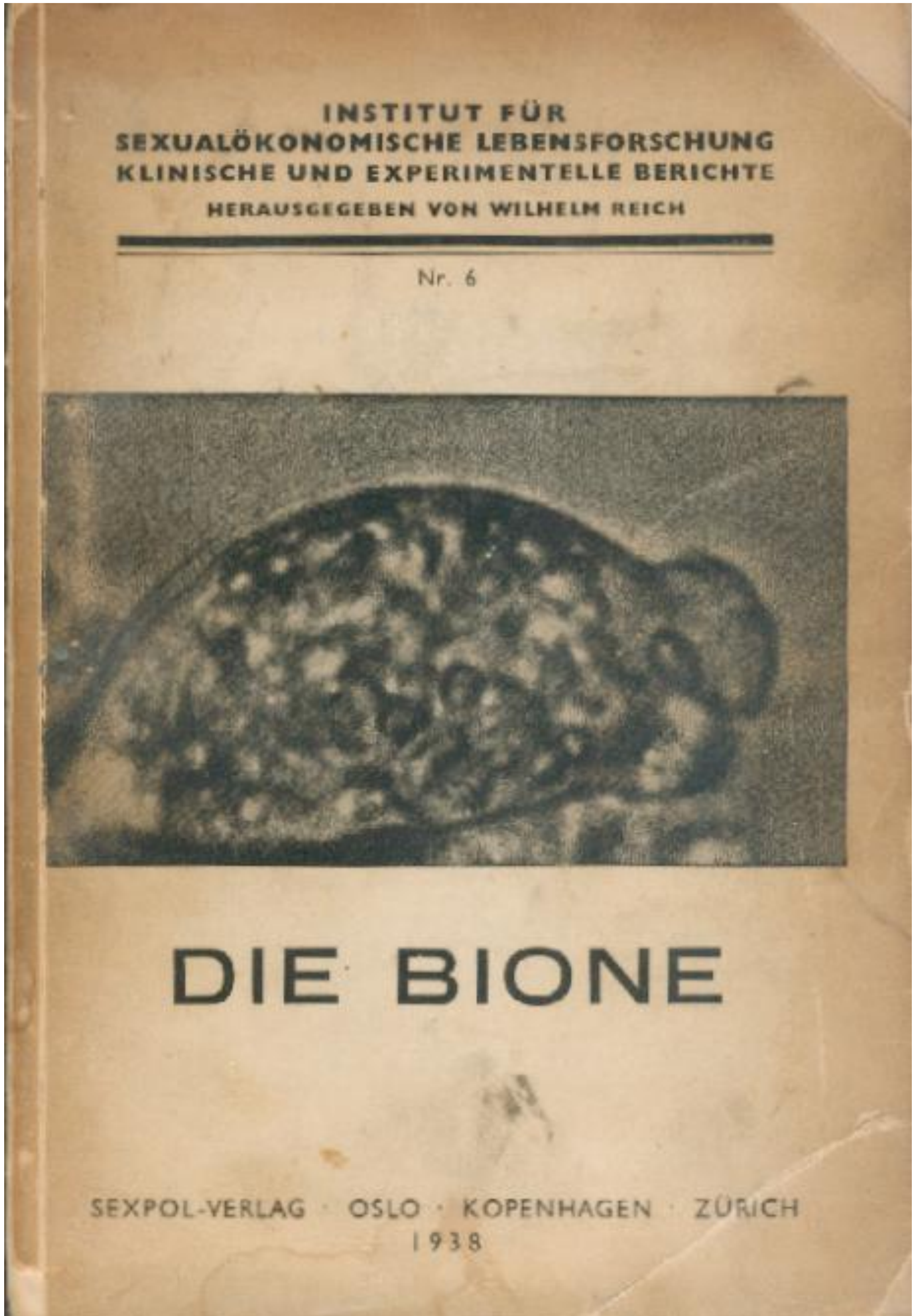


dual photon mass now due to magnetic "bions"-
leading cause of confinement in SU(N) with adjoints at small L
(incl. SYM)

$$e^{-2S_0}\left(e^{2i\sigma} + e^{-2i\sigma}\right)$$ - "bion" operators

turns out "bions" are also not exactly new... (courtesy A. Kronfeld)



"Bions are transitional forms between non-living and living matter. The bion is the elemental functioning unit of all living matter. At the same time, it is the bearer of a quantum of orgone energy and, as such, functions in a specifically biological way."

Wilhelm Reich (1897-1957)
- father of "orgone theory"

to summarize, in QCD(adj),

M + KK* = B - magnetic "bions" -
carry magnetic charge
no topological charge (non self-dual)
(locally 4d nature crucial: no KK in 4d)
generate "Debye" mass for dual photon

main tools

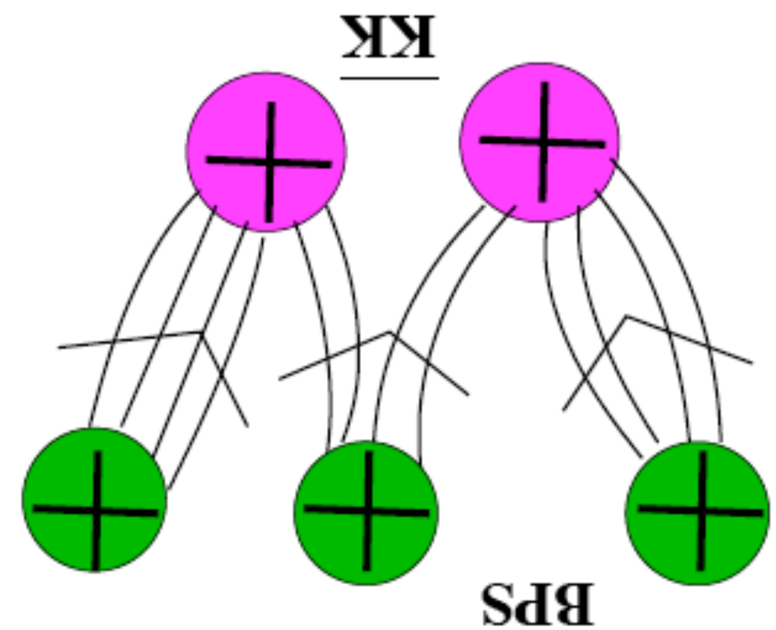
- intertwining of topological shift symmetry & chiral symmetry
- index theorem

topological objects generating magnetic screening depend on massless fermion content - not usually (?) thought that fermions relevant

using these tools, one can analyze any theory...

to summarize, in QCD(adj),

cartoon of a "quintet":



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using these tools, one can analyze any theory...

e.g., in the last couple of years:

name codes: U=Unsal S=Shifman Y=Yaffe P=the speaker

vectorlike | chiral

Theory all SU(N)	Confinement mechanism on $\mathbb{R}^3 \times S^1$	Index for monopoles $[\mathcal{I}_1, \mathcal{I}_2, \ldots, \mathcal{I}_N]$ Nye-M.Singer '00; PU '08	Index for instanton $I_{inst.} = \sum_{i=1}^{N} I_i$ Atiyah-Singer	(Mass Gap)^2 units ~1/L^2
YM Y,U '08	monopoles	$[0, \ldots, 0]$	0	e^{-S_0}
QCD(F) S,U '08	monopoles	$[2, 0, \ldots, 0]$	2	e^{-S_0}
SYM /QCD(Adj) U '07	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(BF) S,U '08	magnetic bions	$[2, 2, \ldots, 2]$	$2N$	e^{-2S_0}
QCD(AS) S,U '08	bions and monopoles	$[2, 2, \ldots, 2, 0, 0]$	$2N - 4$	e^{-2S_0}, e^{-S_0}
QCD(S) P,U '09	bions and triplets	$[2, 2, \ldots, 2, 4, 4]$	$2N + 4$	e^{-2S_0}, e^{-3S_0}
SU(2)YM $I = \frac{3}{2}$ P,U '09	magnetic quintets	$[4, 6]$ SUSY version: ISS(henker) model of SUSY [non-]breaking	10	e^{-5S_0}
chiral $[SU(N)]^K$ S,U '08	magnetic bions	$[2, 2, , \ldots, 2]$	$2N$	e^{-2S_0}
$AS + (N-4)\overline{F}$ S,U '08	bions and a monopole	$[1, 1, , \ldots, 1, 0, 0]$ + $[0, 0, \ldots, 0, N-4, 0]$	$(N-2)AS + (N-4)\overline{F}$	e^{-2S_0}, e^{-S_0},
$S + (N+4)\overline{F}$ P,U '09	bions and triplets	$[1, 1, , \ldots, 1, 2, 2]$ + $[0, 0, \ldots, 0, N+4, 0]$	$(N+2)S + (N+4)\overline{F}$	e^{-2S_0}, e^{-3S_0},

Table 1. Topological excitations which determine the mass gap for gauge fluctuations and chiral symmetry realization in vectorlike and chiral gauge theories on $\mathbf{R}^3 \times \mathbf{S}^1$. Unless indicated otherwise,

+ SO(N),SP(N) - S. Golkar 0909.2838; for mixed-representation/higher-index reps. SU(N) - P,U 0910.1245

So, I have now introduced all the key players:

3d Polyakov model & "monopole-instanton"-induced confinement — Polyakov, 1977

"monopole-instantons" on $R^3 \times S^1$ — K. Lee, P. Yi, 1997; P. van Baal, 1998

the relevant index theorem — Nye, Singer, 2000; Unsal, EP, 2008

center-symmetry on $R^3 \times S^1$ - adjoint fermions or double-trace deformations — Shifman, Unsal, 2008; Unsal, Yaffe, 2008

"bions", "triplets", "quintets"... - new non-self-dual topological excitations and confinement — Unsal, 2007; Unsal, EP, 2009

The upshot is the **dual lagrangian of QCD(adj)** on a circle of size L:

$$\frac{g_4^2(L)}{2L}(\partial\sigma)^2 - \frac{b}{L^3}\, e^{-2S_0}\cos 2\sigma + i\bar{\lambda}^I\gamma_\mu\partial_\mu\lambda_I + \frac{c}{L^{3-2N_f}}\, e^{-S_0}\cos\sigma(\det_{I,J}\lambda^I\lambda^J + \text{c.c.})$$

B, B*

M, KK+*

leading-order perturbation theory; perturbative corrections ~ $g_4(L)^2$ omitted

$$m_\sigma \sim \frac{1}{L}e^{-S_0} = \frac{1}{L}e^{-\frac{8\pi^2}{N_c g_4^2(L)}}$$

$$(\Lambda L)^{\beta_0} = e^{-\frac{8\pi^2}{g_4^2(L)}}$$

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}N_w N_c$$

$$m_\sigma = \frac{1}{L}(\Lambda L)^{\frac{\beta_0}{N_c}} = \Lambda(\Lambda L)^{\frac{\beta_0}{N_c}-1} = \Lambda(\Lambda L)^{\frac{8-2N_w}{3}}$$

mass gap ~ string tension behaves in an interesting way

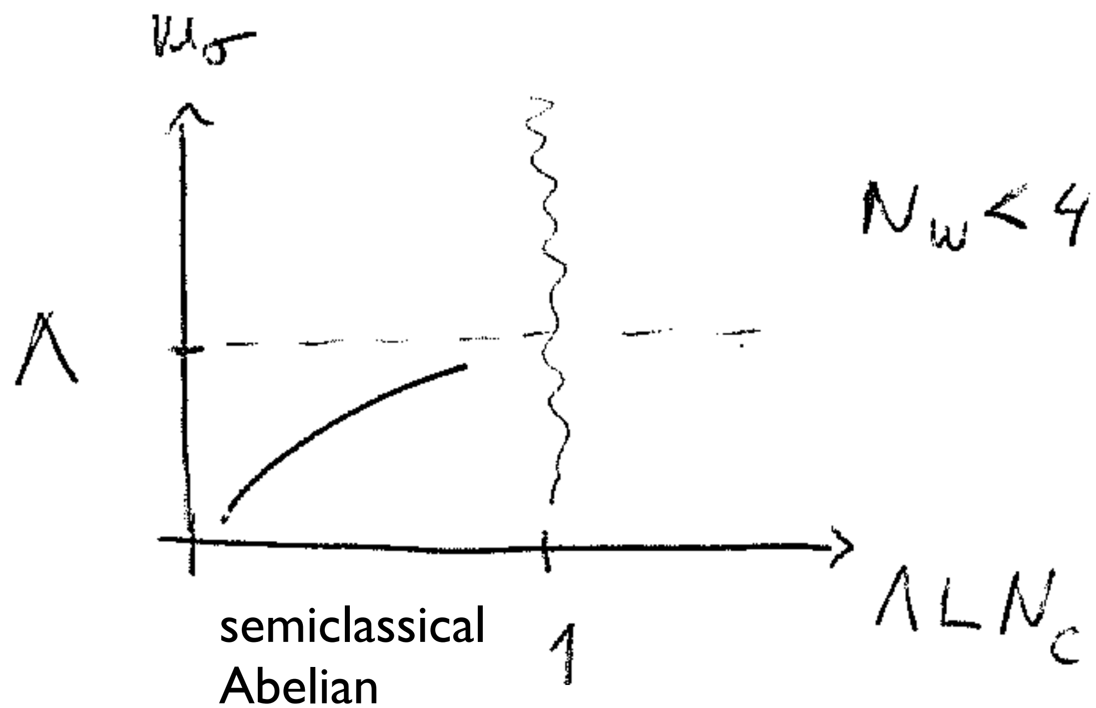
as L changes at fixed Λ ... $N_w^* = 4$?

region of validity of semiclassical analysis:

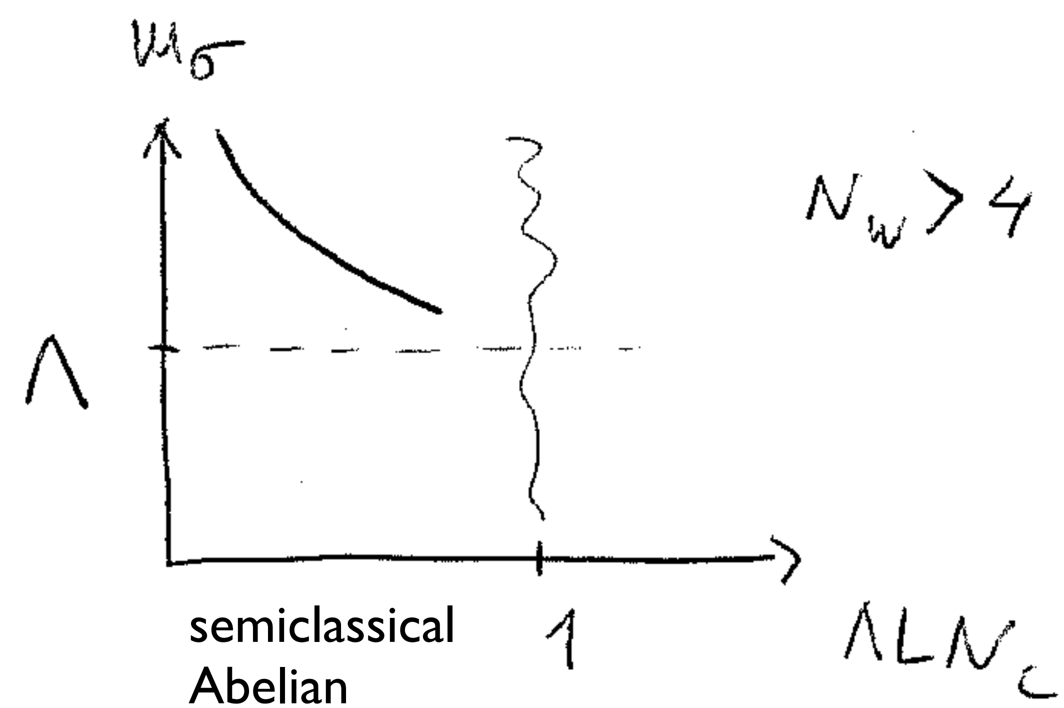
$$\Lambda L \ll 1 \qquad (N_c \Lambda L \ll 1 \text{, really})$$

as mass of W~1/(NL): volume independence (Catterall's talk) and semiclassical (this talk) do not overlap!

$$m_\sigma \sim \Lambda (\Lambda L)^{(8-2N_w)/3}$$



semiclassical
Abelian
confinement



semiclassical
Abelian
confinement

analysis shows that this switch of behavior as number of fermion species is increased occurs in all theories - vectorlike or chiral alike

in each case we obtain a value for the critical number of "flavors" or "generations"... N_f^*

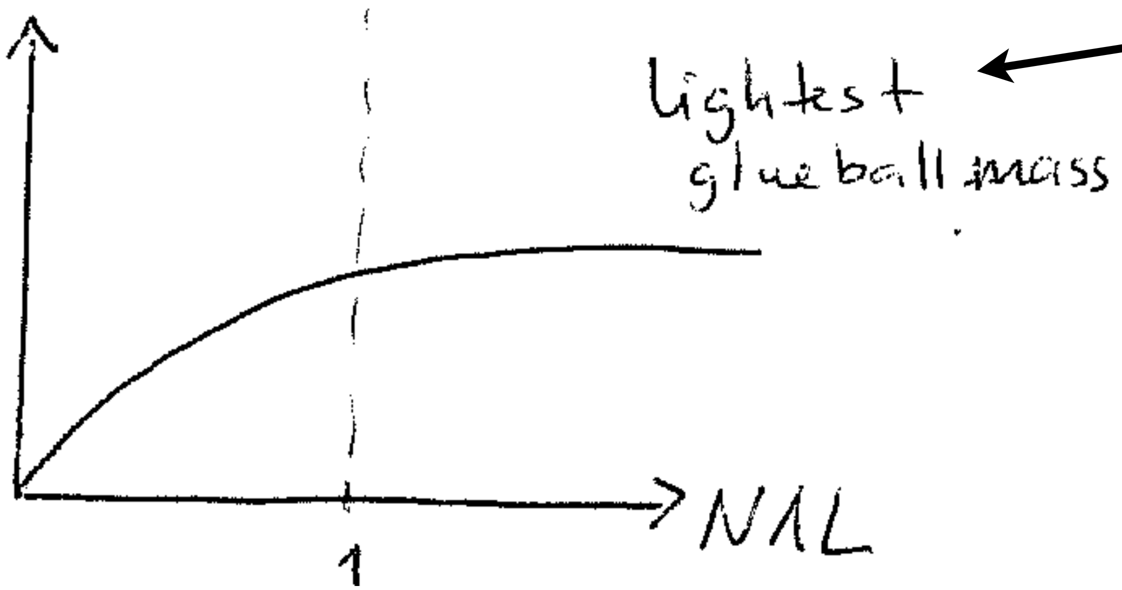
like $N_w^* = 4$ for QCD(adj)

does it tell us anything about R^4?

I know I am in danger of being arrested...

... how **dare** you study non-protected quantities?

A reasonable expectation of what happens at very small or very large number of "flavors" is this:



topological excitations become non-dilute with increase of L, cause confinement,
M, KK+* operators

$$e^{-S_0} \cos\sigma(\det_{I,J} \lambda^I \lambda^J + \text{c.c.})$$

become strong, can cause chiral symmetry breaking (whenever the confining theories break their nonabelian chiral symmetries)

sufficiently small # fermion species
confining theories

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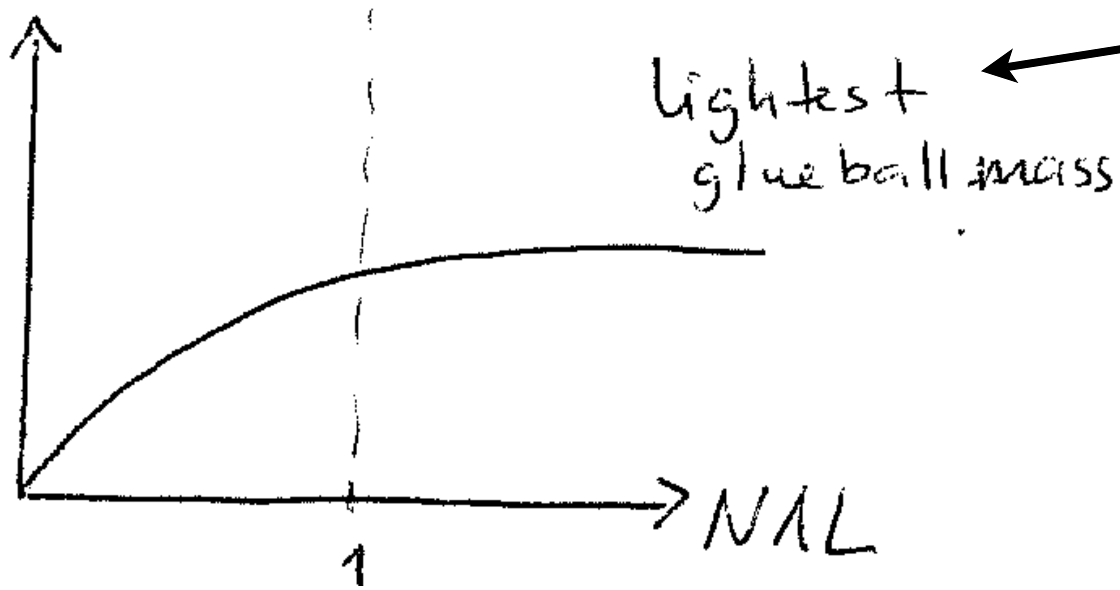

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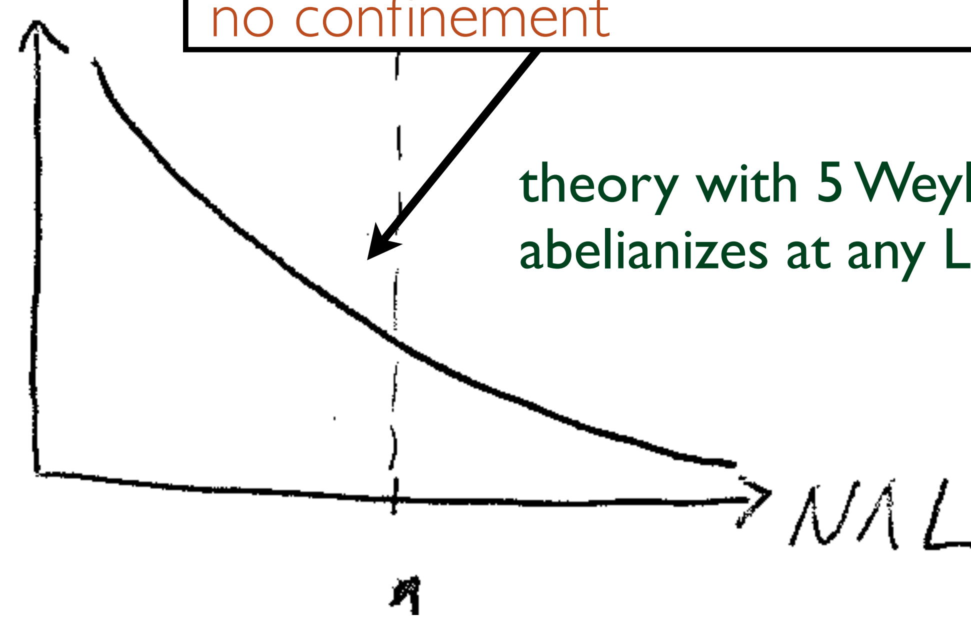
sufficiently small # fermion species
confining theories

sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

topological excitations that cause confinement dilute with increase of L, no confinement



theory with 5 Weyl adjoints abelianizes at any L

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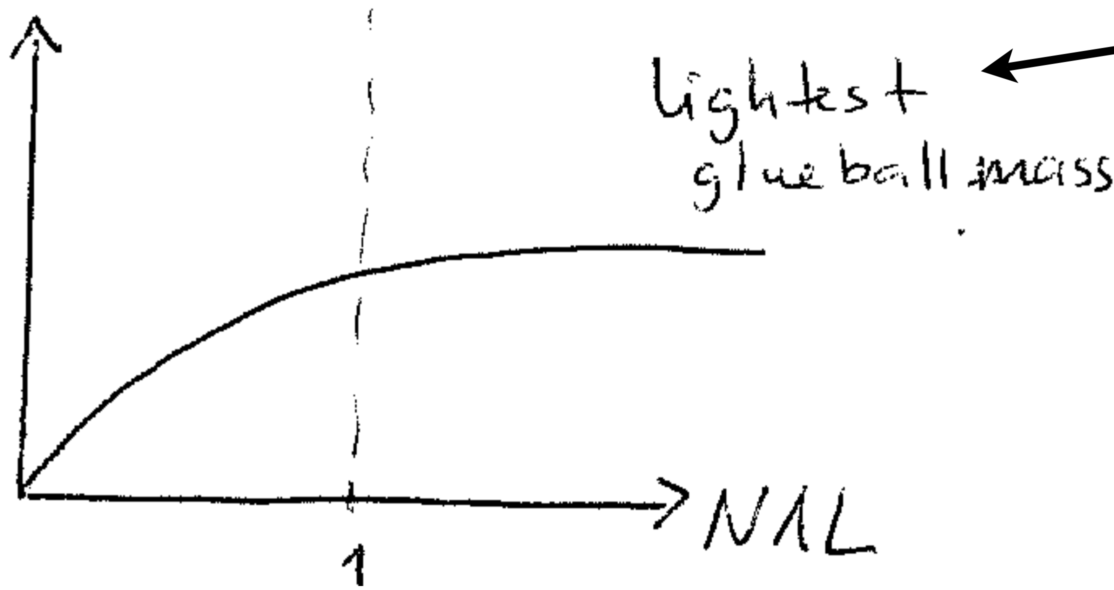

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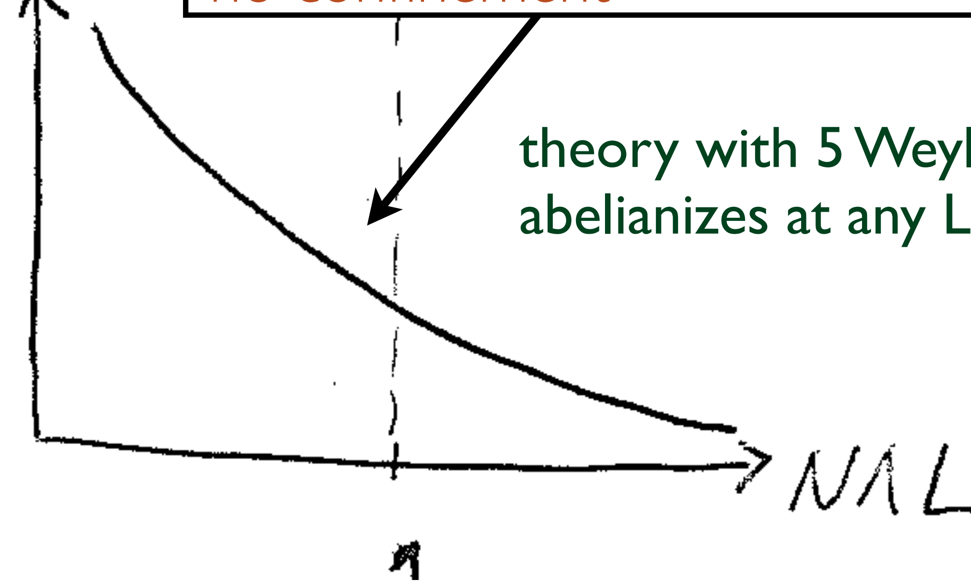


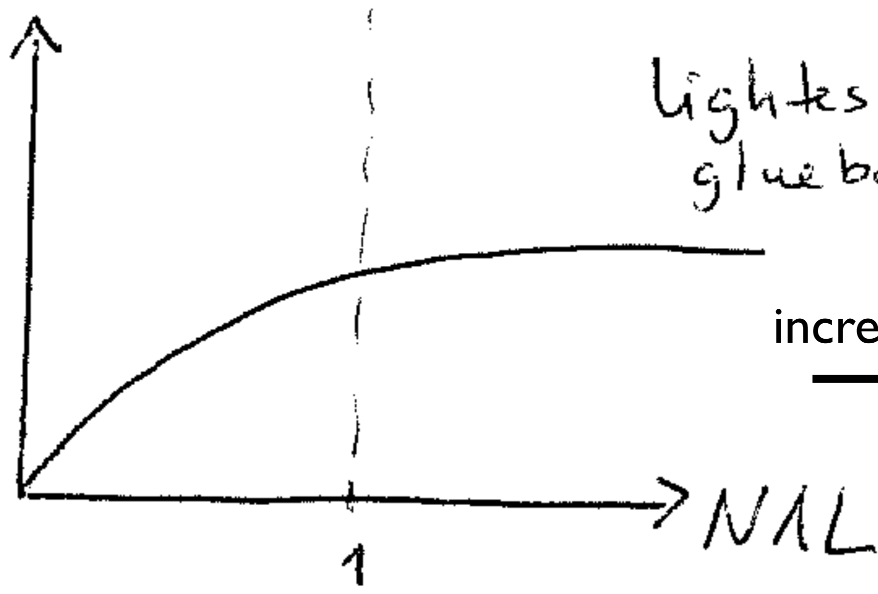
theory with 5 Weyl adjoints abelianizes at any L

but where does the transition **really** occur?
is it at our value N_f*?

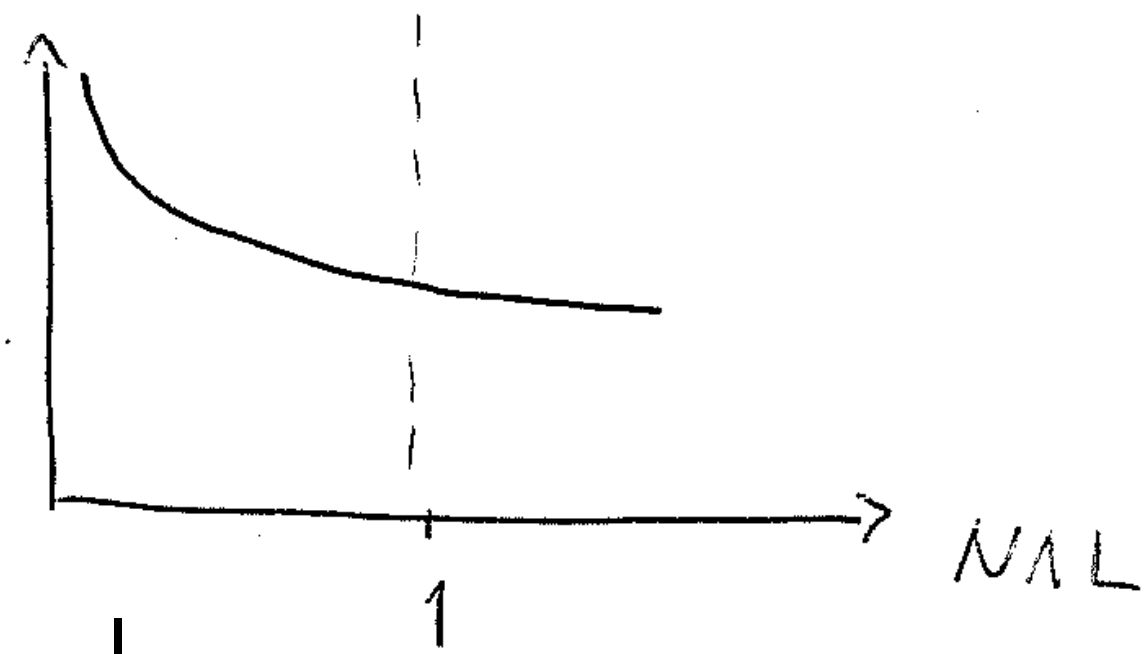
there appear to be three possibilities
(in any given class of theories, only one realized)

A.) our N_f* is the true critical value N_{crit} [theory that may be in this class: QCD(adj), experiment (lattice)]

B.) if, as # species is increased above N_f^*



increase # fermions →



sufficiently small # fermion species
confining theories

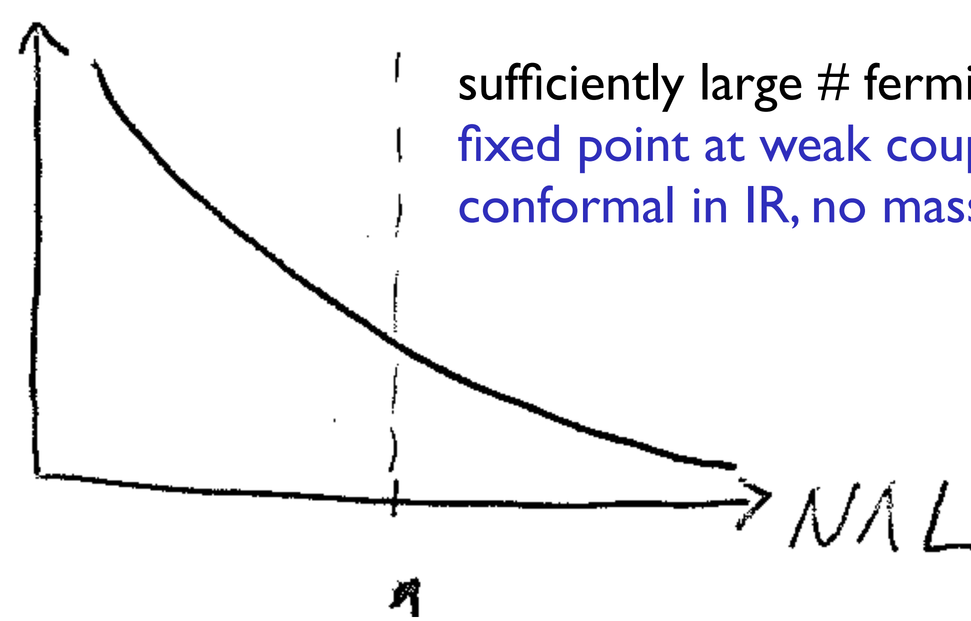
↓ increase # fermions

then, $N_{crit} > N_f^*$

↑

true value of critical # "flavors"

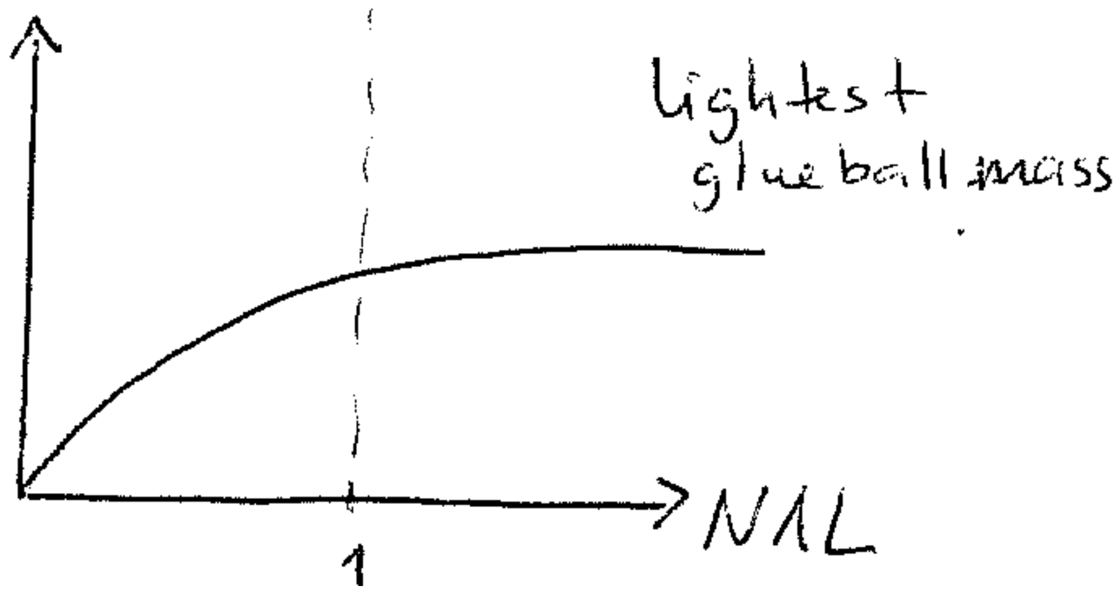
thus, for such theories N_f^* is a lower bound thereof



sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

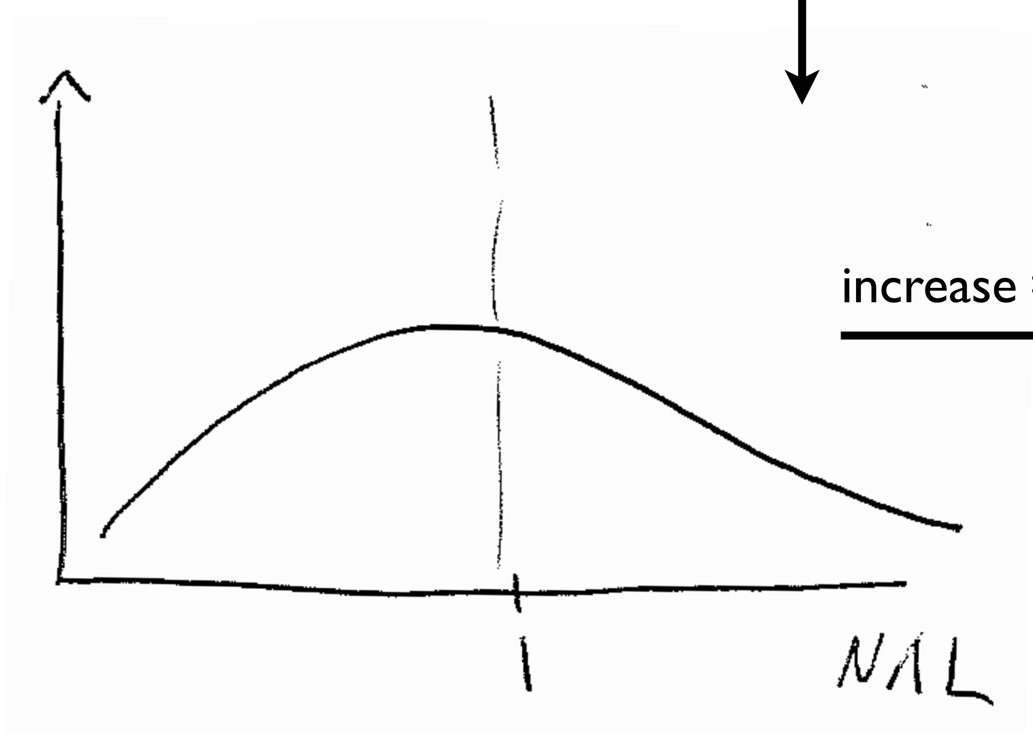
[theory believed to be in this class: QCD(F) - arguments using mixed reps., experiment (lattice)]

C.) if, as # species has not yet reached N_f^*

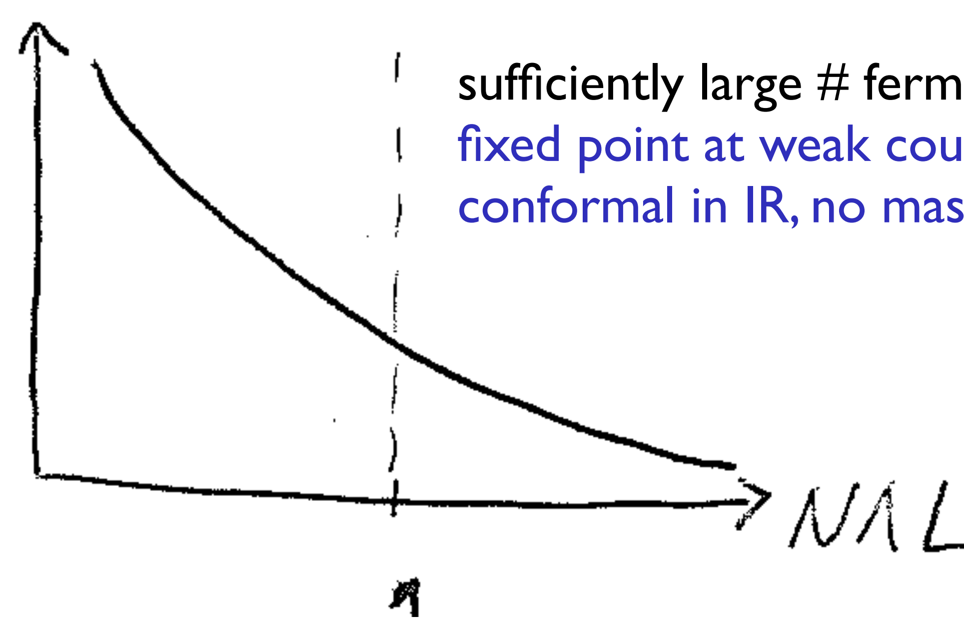


sufficiently small # fermion species
confining theories

increase # fermions



increase # fermions



sufficiently large # fermion species
fixed point at weak coupling
conformal in IR, no mass gap

then,

$N_{crit} < N_f^*$

thus, for this class of theories N_f^* is an upper bound on critical # "flavors"

[only one theory we know is believed to be in this class: SU(2) 4-index symmetric tensor Weyl, theory arguments]

comparing theory estimates of critical number of fermions for SU(N)

"experiment"

Weyl adjoints [no deformation needed]

	our estimate	gap eqn	beta function gamma=2/1	AF lost
any N	4	4.15	2.75/3.66	5.5

4 **?** e.g.: *Catterall et al; del Debbio, Patella,Pica (+et al); Hietanen et al.*

Dirac 2-index (anti)symmetric tensor [deformation needed; but large-N equiv!]

N	our estimate	gap eqn	beta function gamma=2/1	AF lost
3	2.40	2.50	1.65/2.2	3.30
4	2.66	2.78	1.83/2.44	3.66
5	2.85	2.97	1.96/2.62	3.92
10	3.33	3.47	2.29/3.05	4.58
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Dirac fundamentals [deformation needed]

N	our estimate **A/B**	gap eqn	functional RG	beta function gamma=2/1	AF lost
2	5/8	7.85	8.25	5.5/7.33	11
3	7.5/12	11.91	10	8.25/11	16.5
4	10/16	15.93	13.5	11/14.66	22
5	12.5/20	19.95	16.25	13.75/18.33	27.5
10	25/40	39.97	n/a	27.5/36.66	55
∞	$2.5N/4N$	$4N$	$\sim(2.75-3.25)N$	$2.75N/3.66N$	$5.5N$

12 ? e.g.: *Appelquist,Fleming, Neal; Deuzemann, Lombardo,Pallante; Iwasaki et al; Fodor et al; Jin, Mahwinney; A. Hasenfratz*

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- gap equation and lattice - only vectorlike theories
- in chiral gauge theories with multiple "generations" our estimates were the only known ones until Sannino's recent 0911.0931 via the proposed exact beta function

this - largely (given the absence of credible error bars) - agreement is, to us, somewhat amusing...

let's compare the tools used:

gap equation *Appelquist et al; Miransky et al; Ryttov, Sannino (FRG: Gies, Jaeckel)*	conformality tied to absence of chiral symmetry breaking **compares fixed-point coupling to critical gauge coupling for chiral symmetry breaking** *-ladder diagram "approximation" of truncated Schwinger-Dyson eqns. for fermion propagator in Landau gauge* *-must use at least 2-loop beta function to get fixed point value of g*
beta function *Ryttov, Sannino*	loss of conformality tied to anomalous dimension of fermion bilinear at IR fixed point violating unitarity bound (or close to it) **postulate exact beta function**
our "estimate"	conformality tied to absence of mass gap/string tension see also Armoni, 2009 (worldine approach; very similar numbers) **semiclassical analysis on a non-thermal circle** **dilution vs. non-dilution of topological excitations with L**
lattice	in principle, a first-principle determination (modulo V, m, a)

Conclusions 1:

Compactifying 4d gauge theories on a small circle is a "deformation" where nonperturbative dynamics is under control - dynamics as "friendly" as in SUSY, e.g. Seiberg-Witten.

Confinement is due to various "oddball" topological excitations, in most theories non-self-dual.

Polyakov's "Debye screening" mechanism works on $R^3 \times S^1$ also with massless fermions, contrary to what many thought - KK monopoles and index theorem-crucial ingredients of analysis.

Precise nature - monopoles, bions, triplets, or quintets - depends on the light fermion content of the theory.

U,P; 0812.2085, 0906.5156

Conclusions II:

didn't have time for these:

Found chiral symmetry breaking (Abelian) due to expectation values of topological "disorder" operators: occurs in mixed-rep. theories with anomaly-free chiral U(1), broken at any radius

U,P; 0910.1245

Circle compactification gives another calculable deformation of SUSY theories - not yet fully explored -

in l=3/2 SU(2) Intriligator-Seiberg-Shenker model we argued that theory conformal, rather than SUSY-breaking.

U,P; 0905.0634
agreement with
different arguments of
Shifman, Vainshtein `98
Intriligator `05

Conclusions III:

Gave “estimates” of conformal window boundary in vectorlike and chiral gauge theories (~OK with “experiment” when available).

Conformality tied to relevance vs irrelevance of topological excitations.

U,P; 0906.5156

Further similarity to KT transition? Kaplan, Son, Stephanov, 2009
- in 2d, vortices proliferate in high-T phase and irrelevant in conformal phase of the XY model

Clearly, on R^3x S^1 we only see the “shadow” of the real thing...

Conclusions IV: Questions?

Is it so crazy to expect "relevance vs. irrelevance" (with changing Nf) of topological excitations also in R^4?

Lattice studies in pure YM (early ref.: Kronfeld et al, 1987) have found that confinement appears to be due to topological excitations- center vortices, monopoles – these are 't Hooft's (1978) "transient particles" that are revealed to us in particular gauges – and the deconfinement transition at high-T is associated with them becoming irrelevant.
(huge body of literature & apparently not much agreement in the details...)

To expect that massless fermions would affect the nature of topological excitations is thus quite natural.

What is harder (for me?) is how to make this precise on R^4.

Conclusions V: Questions?

Back to SUSY? - theorists' "safe haven"

We argued that "bions" are responsible for confinement in N=1 SYM at small L (a particular case of our Weyl adjoint theory).
This remains true if N=1 obtained from N=2 by soft breaking

Monopoles and dyons are responsible for confinement in N=2 softly broken to N=1 at large L. (Seiberg, Witten `94)

So, in different regimes we have different pictures of confinement in N=1 SYM.

Do they connect in an interesting way?

e.g., recent works Gaiotto, Moore, Neitzke `09; Chen, Dorey, Petunin `10
on "wall-crossing" at finite L?

back-up

Found chiral symmetry breaking (Abelian) due to expectation values of topological "disorder" operators.

- didn't have time for this (occurs in mixed-rep. theories)

Example:

Unsal,EP 0910.1245

$(N_{\rm adj}{=}1,\ N_{\rm F}{=}1)\ SU(2)$

	$U(1)_B$	$U(1)_A$
λ	0	1
ψ_L	1	-2
ψ_R	-1	-2
$e^{i\sigma}$	0	-2

$(N_{\rm adj}, N_{\rm F}) = (1,1)$:

small-L: disorder operator vev
Goldstone is dual photon

$$\mathcal{L}^{\rm dual} \supset e^{-S_0}\langle e^{i\sigma}\rangle\lambda^2 + e^{-2S_0}\langle e^{-2i\sigma}\rangle\psi_L\psi_R$$

$$\langle\lambda\lambda\rangle = \Lambda^3 e^{i\pi/f_\pi}, \qquad \langle\psi_L\psi_R\rangle = \Lambda^3 e^{-2i\pi/f_\pi}$$

large-L: fermion condensate
Goldstone is "pion-like"

small-L and large-L regimes can smoothly merge via NJL-like breaking due to monopole operators becoming strong at L ~ Lambda

A_4 - adjoint 3d scalar Higgs field;
a gauge-covariant description:

$$W = P e^{i \oint_{S_1} A_4 dx^4}$$

"holonomy" around circle (or "Polyakov loop")
- a unitary gauge-group element
- eigenvalues lie on unit circle
- trace of Polyakov loop is gauge invariant

if the expectation values are

$$\langle W \rangle = \begin{pmatrix} e^{i\pi/2} & \\ & e^{-i\pi/2} \end{pmatrix}$$

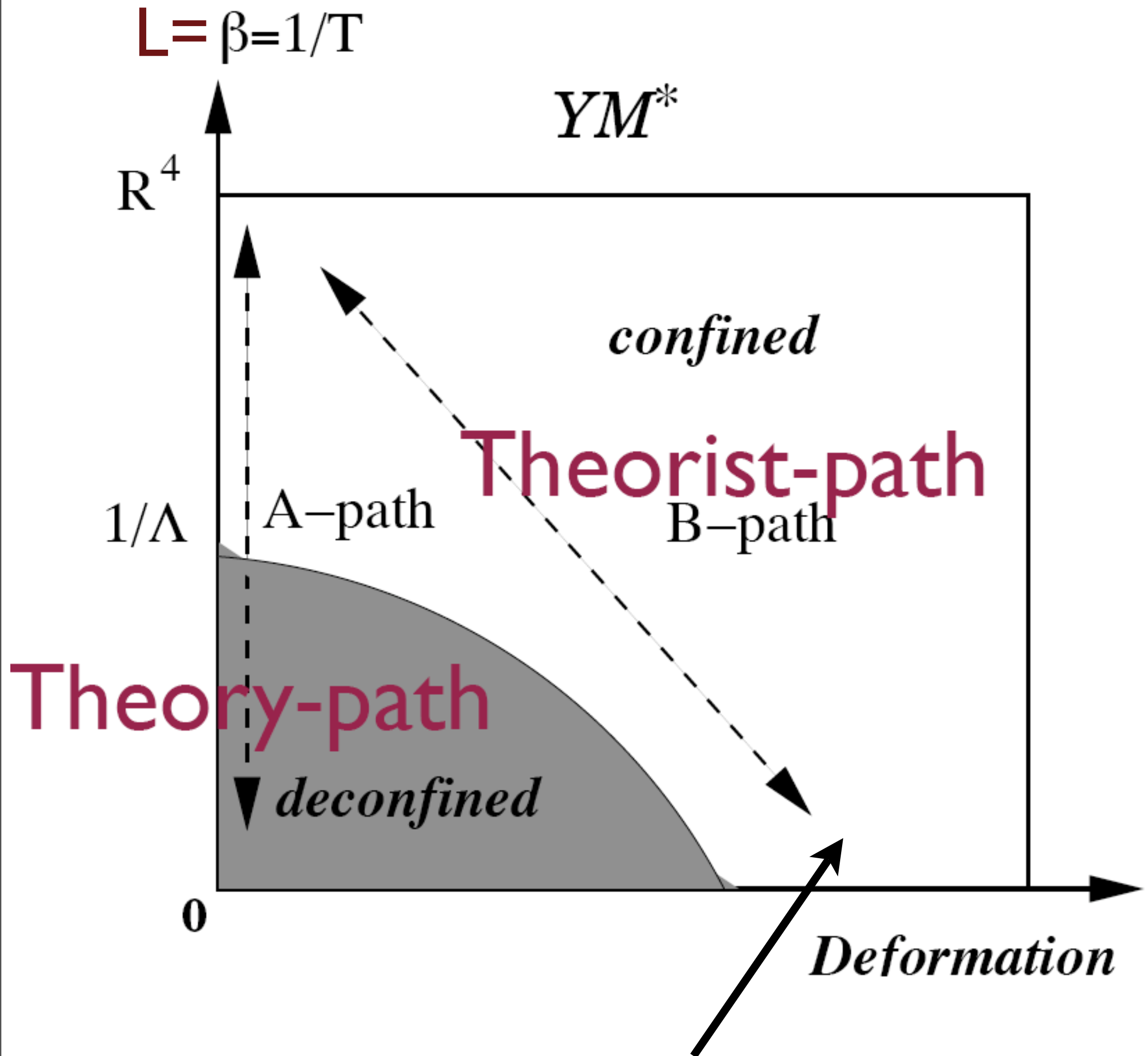
then $\mathrm{tr}\langle W \rangle = 0$

and we say that *"center symmetry is preserved"*

$\mathrm{tr}\, W \rightarrow e^{i\pi}\, \mathrm{tr}\, W$ for SU(N): $e^{i\frac{2\pi}{N}}$

we are interested in **unbroken center**:
where <trW>=0 and SU(2) broken to U(1) at scale 1/L

II.:"Deformation Theory"-needed, e.g., in QCD with fundamentals, not needed in QCD with adjoints, SUSY, etc...



theory is under control here:

can calculate mass gap for gauge fluctuations, string tensions (as in Seiberg-Witten theory)

decompactification smooth in the sense of center symmetry

$$S^{\mathrm{YM}^*} = S^{\mathrm{YM}} + \int_{R^3 \times S^1} P[U(\mathbf{x})]$$

$$P[U] = A \frac{2}{\pi^2 \beta^4} \sum_{n=1}^{\lfloor N/2 \rfloor} \frac{1}{n^4} \left|\mathrm{tr}\,(U^n)\right|^2$$

L

double-trace deformations

- lattice studies back up smoothness in some models (Ogilvie, Myers, Meisinger, 2008)
- interesting at large-N: P[U] ensures center symmetry but decouples from observables... in the volume-independence context (Unsal, Yaffe, 2008)

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in what follows, we assume center-symmetric vacuum - due to either I. or II.