Minimal walking on the lattice - a status report and some new ideas

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(Walking) technicolor

MWT on the lattice

Phase diagram, spectrum, evidence for IRFP

Large N reduction

Monte Carlo Renormalization Group

Troubles with Higgs

Standard model Higgs has many problems:

- Quadratic unstability $m_H \sim O(\Lambda)$
- Triviality – quartic coupling $\lambda \to 0$ as $\Lambda \to \infty$
- Symmetry breaking “put in by hand” - what sets scale $\Lambda_{EW} << \Lambda_{GUT}$?

One attractive (and conservative) solution:

- Dispense with Higgs
- Invoke new strong dynamics to break EW symmetry: technicolor

Basic idea

- Assume new strong dynamics at scale $O(\Lambda_{TC}) \sim 1\mathrm{TeV}$
- Causes condensation of new techniquarks
- If techniquarks carry EW quantum numbers – breaks $SU(2) \times U(1)$
- Massive W,Z by eating would be Goldstones from spontaneous breaking techniquark chiral symmetries.

Advantages:

- Follows example of QCD - $\Lambda_{EW} << \Lambda_{GUT}$ natural.
- No quadratic instability or triviality.

Problems

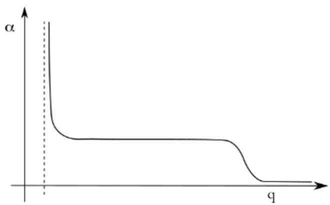
- How many techniquarks, how many colors, what representation ?
- Phenomena arise thru strong dynamics: how to make predictions ?
- Masses quarks/leptons arise from coupling techniquarks to SM particles in extended ETC theory. What determines flavor hierarchy, symmetry breaking at high scales ?

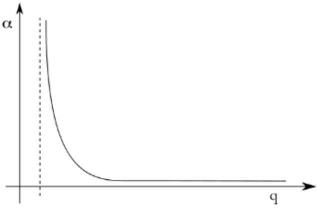
Why QCD won't do

- Scaled up QCD at odds with precision EW observables (S parameter)
- Generic ETC theories - 4 quark ops at $E < \Lambda_{ETC}$ yield FCNC. Suppress by large Λ_{ETC}. But then SM masses too small!

Dynamics must be different from QCD: walking ?

Walking





- Slow running implies near conformal
- S parameter not given by QCD.
- Condensate enhanced - large anomalous dim $< \overline{\psi}\psi >$ alleviates small mass problem

Minimal walking theory

- 2 flavors of adjoint fermions in SU(2)
- Ladder approx/all orders β-function suggest conformal for $N_f \gtrsim 2$. Good for EW precision.
- Lattice potentially offers way to determine whether or not theory is conformal at large distances
- If theory walks – use lattice to calc. technihadron spectrum, S parameter, condensate etc

MWT on lattice: gluons

Employ standard Wilson gauge action

$$S_G = -\frac{\beta}{2}\sum_x \sum_{\mu>\nu} \mathrm{ReTr}\left(U_\mu(x)U_\nu(x+\hat{\mu})U^\dagger_\mu(x+\hat{\nu})U^\dagger_\nu(x)\right)$$

where

- $U_\mu(x) = e^{A_\mu(x)a}$ gauge link – $SU(2)$
- $\beta = \frac{4}{g^2}$
- Can show $S_G \to \int d^4x\, F^2_{\mu\nu} + O(a)$

MWT on lattice: fermions

Wilson techniquarks:

$$S_F = \sum_{x,\mu} \overline{\psi}(x) \left(V_\mu(x) (I - \gamma_\mu) \psi(x+\mu) + V^T_\mu(x-\mu) (I+\gamma_\mu) \psi(x-\right.$$
$$- \sum_x (2m+8) \overline{\psi}(x)\psi(x)$$

with adjoint links

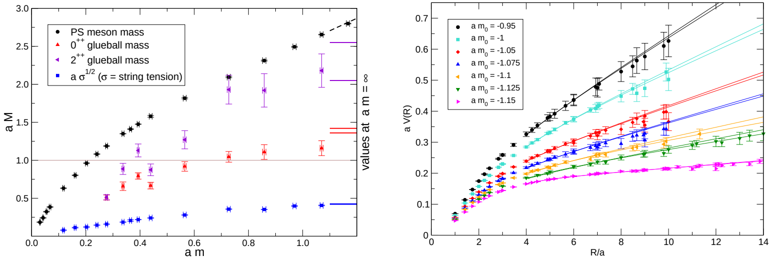
$$V^{ab}_\mu(x) = \mathrm{Tr}\left(\sigma^a U_\mu(x) \sigma^b U^\dagger_\mu(x)\right)$$

Drawbacks: need to tune bare quark mass m with β to restore chiral symmetry.

Results of numerical simulations

- Studied by several groups using lattices $(8-24)^3 \times 32(64)$
- Bulk phase transition $\beta_B \sim 2.0$. For $\beta > \beta_B$ (continuum phase):
  - Find $m_\pi / m_\rho \sim \mathrm{const}$ as $m_q \to 0$ independent of β
  - String tension (very) small
  - Glueballs light $m_G / m_\pi << 1$
  - $f_\pi \to 0 \quad L \to \infty$
  - $< \overline{\psi}\psi > \to 0 \quad m_q \to 0$

Spectrum and static potential



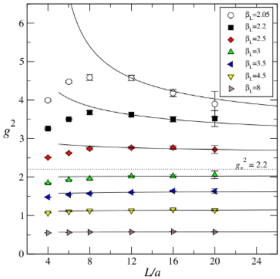
L. Del Debbio, B. Lucini, A. Patella, C. Pica, A Rago, arXiv:1004.3197

Consistent with IRFP.

Note that $m_\pi (m_G)/m_\rho \rightarrow \mathrm{const} \quad m \rightarrow 0$

Interpretation as mass deformed CFT.

Running of coupling



A. Hietanen, K. Rummukainen, K. Tuominen, PRD80:094504, 2009.

- Very slow.
- Backward running at strong (bare) coupling.

Conclusion

Significant evidence for IRFP in MWT

BUT

How to be sure ...?

- What if our lattices are just too small ?
- Perhaps theory walks with confinement scale very large ?
- Really want methods to handle finite volume effects ?

Large N reduction

- 1982 Eguchi and Kawai proposed that observables in SU(N) gauge theories become independent of volume as $N \to \infty$.
- Implies that can calculate physical quantities on infinite lattices from the single site theory in large N limit!
- For SU(N) coupled to adj fermions critical number of flavors for conformality independent of N
- Hence remove finite volume effects from conformality question in MWT by keeping small volume but going to large N.

Not so fast ...

- EK's volume reduction proof requires the theory satisfy a number of constraints.
- Most important of these relates to a property of the lattice theory called Z_N center symmetry
- For D dimensional $SU(N)$ theory center symmetry transformation

$$U_\mu \to U_\mu z_\mu^n \quad \mu = 1 \dots D$$

where $z = e^{\frac{2\pi i}{N}}$ and $n_\mu = 0 \dots N-1$.

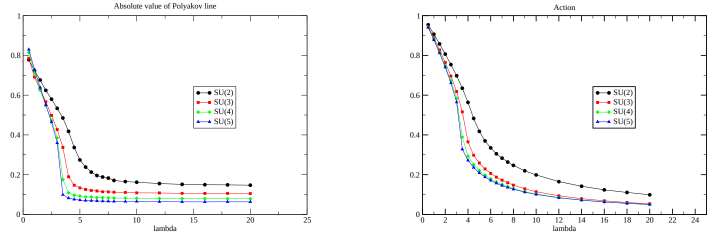
- Symmetry of pure gauge theory and also with adjoint fermions
- Volume reduction possible if center symmetry does not break spontaneously

Check with simulations

- Naive Wilson as before.
- 2^4 lattice:
  - Theoretical considerations favor $L = 2$ rather than single site
  - Practical code issues
  - Maybe no cost – effective $N = N \times L$
- HMC dynamical fermion alg.

with Richard Galvez.

Quenched theory



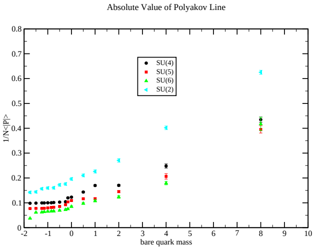
Order parameter for center breaking:
Polyakov line $P = \frac{1}{N} \mathrm{Tr} \prod_{t=0}^{T} U_\mu(t)$

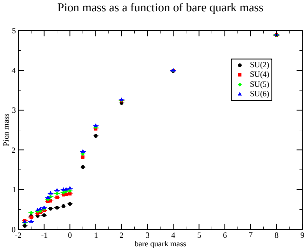
Spontaneous breaking of Z_N

- Bulk (first order) phase transition for $\lambda = \lambda_c \sim 3$ ('t Hooft coupling $\lambda = g^2 N = 2N/\beta$ held fixed as $N \to \infty$).
- If $\lambda > \lambda_c \quad < P > \to 0$ as $N \to \infty$
- But if $\lambda < \lambda_c \quad < P > \to \mathrm{f}(\lambda)$ as $N \to \infty$
- Unfortunately, continuum phase is $\lambda < \lambda_c$!
- In this region – Z_N is broken. No volume reduction for pure gauge theory. Well known.

Add adjoint fermions

Consider $\lambda = 0.5$

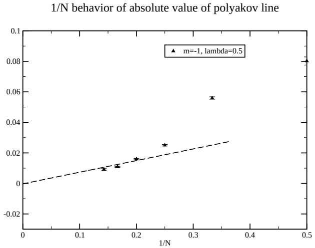




For light quarks $< P > \to 0$ as $N \to \infty$

adj fermions restore center symmetry!

1/N Corrections



Fit quantitatively consistent with vanishing $< P > \rightarrow 0 \quad N \rightarrow \infty$

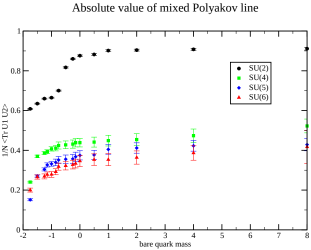
Not actually a surprise

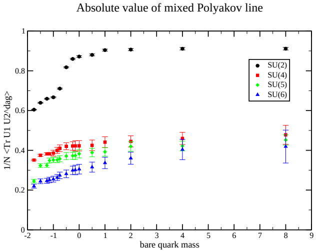
- Consider eigenvalues $e_i, i = 1 \ldots N$ of Polyakov line P
- Kovtun et al showed (continuum) that 1-loop effective potential $V_{\rm eff}(e_i - e_j)$ is repulsive if $N_f(adj) > 0$
- Leads to ${\rm Tr} P = 0$ - center symmetry restoration.
- Generalized to Wilson lattice fermions by Poppitz, Ünsal arXiv:0911.0358

More exotic breakings?

Other breakings of Z_N^D possible corresponding to order parameters

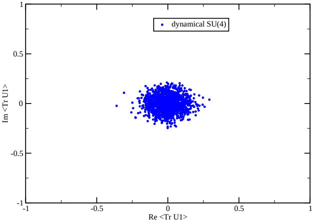
$$P^1_{\mu\nu} = \frac{1}{N}\mathrm{Tr}(U_\mu U_\nu)$$
$$P^2_{\mu\nu} = \frac{1}{N}\mathrm{Tr}(U_\mu U_\nu^\dagger)$$

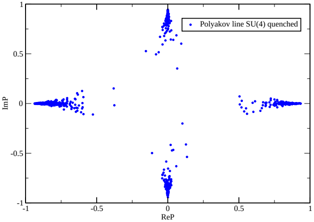




Scatter plot of P

$\lambda = 0.5\ SU(4)$:





$N_f = 2adj\ m = -1$

quenched

Z_N symmetry in large N limit

In progress

- Compute meson spectrum, string tension, vs N on $2^3 \times T$ lattices.
- Compare with 1 flavor case (shown by Bringoltz et al to be Z_N symmetric)
- Can we see qualitative/quantitative signals for conformality ?

Monte Carlo Renormalization Group

- Generate ensemble using MC methods.
- Coarse grain configs a la Wilson - block lattice with fewer dof.
- Determine flow in coupling constants by matching observables with a direct simulation at (same) smaller lattice volume.
  - Repeat analysis by blocking further. After several blocking steps only relevant operators are left and flow approaches renormalized trajectory RT.
  - Optimize blocking step to improve convergence to RT.
- Can see if coupling is indeed irrelevant (as for IRFP)
- Also read off scale dependence of mass - anomalous dim γ.

See Anna Hasenfratz's talk.

Some details - m=0 case

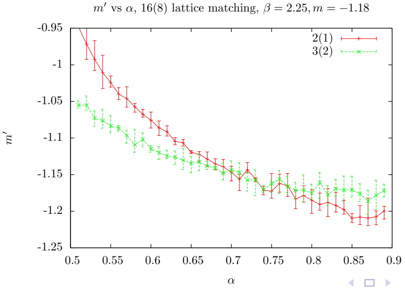
- Take L^4 lattice at some β ($\beta = 2.25\, L = 32$)
- Block by factor of 2 – $L \to L/2$
- Simulate lattice size $L/2$ at range of β'
- Match wilson loops - determine matched β'_m. Same lattice volume – different cut-off.
- Discrete β-function $= (\beta - \beta'_m)$
- Repeat for further blockings
- Similarly for mass flow $m(a/2), m'(a)$ assuming IRFP

with Liam Keegan, Luigi del Debbio.

The block transformation

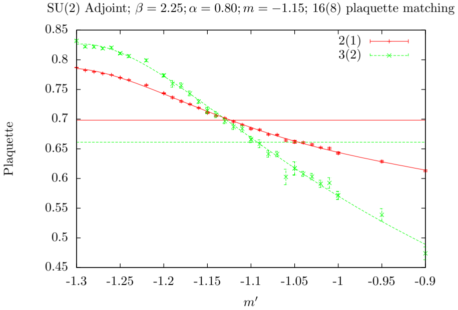
Optimizing RG/block transformation:

$$U^B_\mu(x, x+2\mu) = (1-\alpha)U_\mu(x)U_\mu(x+\mu) + \frac{\alpha}{6}\sum_{\mu,\nu} U_\nu(x)U_\mu(x)U_\mu(x+\mu)U^\dagger_\nu(x+\nu+\mu)$$

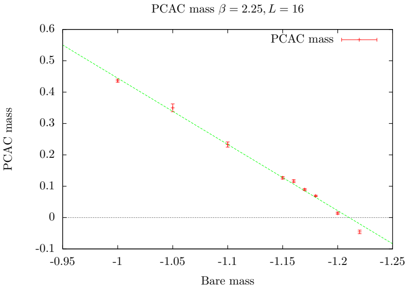


Matching for mass

Match plaquette by varying (bare) quark mass on smaller lattice



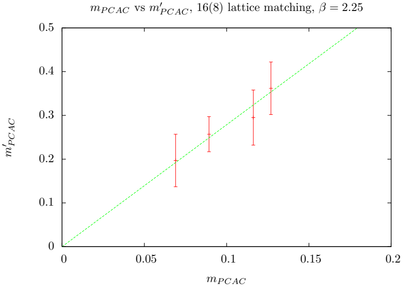
Obtaining the PCAC mass



Convert matching in bare quark mass to matching in PCAC mass.

Anomalous dimension

Prelim result (16^4 lattices)



$$\gamma = 0.48(25)$$

To do ..

- Finish 32^4 lattices. Verify (absence of) flow in $m = 0$ case.
- Higher statistics - errors on γ.
- Several bare couplings - verify true IRFP.
- Better blockings ? (smearing etc) Important for strong coupling.

Conclusions

- MWT theory is example of class of gauge theories currently being studied on lattice – candidates for walking or near walking...
- Conjecture useful for dynamical EW symmetry breaking
- Algorithms, hardware used by lattice QCD applicable.
- Challenging lattice calcs. – (near) conformal invariance broken by cut-off a, non-zero quark mass and finite lattice volume.
- Consensus building that MWT is probably in conformal window - spectrum, $g^2(E)$, ...
- New ideas will be needed for such theories - QCD not always useful. eg. large N. Exploit scale invariance eg MCRG.
- Exciting time for lattice – could play an important role in understanding results from LHC.